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COMPARATIVE CAPABILITIES OF CURRENT RADIOSONDES AND SATELLITES IN DETERMINING GEOPOTENTIAL HEIGHTS AND THEIR SPACE DERIVATIVES AT 300 MB

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1. Introduction

The purpose of this study is to illustrate current and imminent capabilities of radiosondes and satellites in observing the atmosphere. We shall do so by estimating the RMS random errors of gradients and Laplacians of geopotential heights at 300 mb, computed from observations obtained from these sources. We shall perform two sets of computations: one over the United States where the radiosonde observations are statistically homogeneous, and the other over the Atlantic involving different types of instruments. In either case, the present density of observations will be used in the computations. The capabilities of satellites will be based on the quality and number of observations retrieved over the oceans from the present VTPR systems. We shall assume that eventually, the observations retrieved overland will be of comparable quality although we are aware that this may be an optimistic assumption.

In every case, we shall consider only the random errors of observations, assuming that the systematic errors can be corrected.

2. Theory and Procedure

Let h_i denote a set of geopotential heights whose observed values $\hat{h}_i$ are subject to a random error ε_i so that

$$\hat{h}_i = h_i + \varepsilon_i \qquad (i= 1, 2, . . n) \qquad (1)$$

Further, let $\nabla\hat{h}$ denote the computed geopotential gradient whose true value is ∇h. If the gradient is computed from two observations $\hat{h}_1$, $\hat{h}_2$, separated by a distance r,

$$\nabla\hat{h} = \frac{\hat{h}_2 - \hat{h}_1}{r} = \frac{(h_2 + \varepsilon_2) - (h_1 + \varepsilon_1)}{r} \qquad (2)$$

The errors ε_1, ε_2, are known only in a mean square sense and not on any one occasion. We can therefore separate the errors from the true values only by using a statistical procedure. We may do so by assuming that the errors are random, independent of each other and of the true values of the geopotential. From (2), the mean square computed gradient

$$\overline{(\nabla\hat{h})^2} = \overline{\left[\frac{(h_2 + \varepsilon_2) - (h_1 + \varepsilon_1)}{r}\right]^2} \qquad (3)$$

Under the above assumptions, the cross products involving the errors ε_1, ε_2 vanish and we have

$$\overline{(\nabla\hat{h})^2} = \frac{\overline{h_1{}^2} + \overline{h_2{}^2} - 2\,\overline{h_1 h_2}}{r^2} + \frac{\overline{\varepsilon_1{}^2} + \overline{\varepsilon_2{}^2}}{r^2} \qquad (4)$$

The variances $\sigma_1{}^2$, $\sigma_2{}^2$ and the autocorrelation coefficient μ_{12} may be written

$$\sigma_1{}^2 = \overline{(\bar{h} - h_1)^2} = \overline{(\bar{h})^2} - \overline{2h_1\bar{h}} + \overline{h_1{}^2} \qquad (5)$$

$$\sigma_2{}^2 = \overline{(h - h_2)^2} = \overline{(\bar{h})^2} - \overline{2h_2\bar{h}} + \overline{h_2{}^2} \qquad (6)$$

$$\mu_{12} = \frac{\overline{(\bar{h} - h_1)\ (\bar{h} - h_2)}}{\sigma_1\sigma_2} = \frac{\overline{(\bar{h})^2} - \overline{h_1\bar{h}} - \overline{h_2\bar{h}} + \overline{h_1 h_2}}{\sigma_1\sigma_2} \qquad (7)$$

Substituting equations (5), (6), (7) in (4) and assuming that the variance is homogeneous over the region of interest, and the autocorrelation function μ is a function of the distance r only, we have

$$\overline{\nabla \hat{h}} = \frac{2\sigma^2[1-\mu(r)]}{r^2} + \frac{\overline{\varepsilon_1^2} + \overline{\varepsilon_2^2}}{r^2} \tag{8}$$

If the mean square error is homogeneous, as would be the case with radiosonde or satellite observations of the same type, equation (8) becomes

$$\overline{\nabla \hat{h}} = \frac{2\sigma^2[1-\mu(r)]}{r^2} + \frac{2\,\overline{\varepsilon^2}}{r^2} \tag{9}$$

The first term on the right hand side of equations (8) and (9) is the true value of the mean square gradient and the second term is the computational error. If we denote these terms by G and E respectively, we may assess the performance of a given observational system by the noise to signal ratio

$$\eta = \frac{E}{G} \tag{10}$$

For a homogeneous observational system

$$\eta = \frac{\overline{\varepsilon^2}}{\sigma^2[1-\mu(r)]} \tag{11}$$

For a heterogeneous system,

$$\eta = \frac{\overline{\varepsilon_1^2} + \overline{\varepsilon_2^2}}{2\sigma^2[1-\mu(r)]} \tag{12}$$

The corresponding noise to signal ratio for a Laplacian computed from a set of homogeneous observations is

$$\eta' = \frac{5\overline{\varepsilon^2}}{r^2[5-8\mu(r)+2\mu(r\sqrt{2})+\mu(2r)]} \tag{13}$$

To apply equations (8) - (13) we need to know the error statistics of both radiosonde and satellite determinations of geopotential heights, the density of observations, as well as the variance and covariance of geopotential height at 300 mb.

3. Errors in determining geopotential heights

a. Radiosonde errors

There is a certain amount of uncertainty concerning the random errors of radiosonde observations. According to an estimate by Lenhard (1970), based on a set of paired simultaneous AN/GMD-1 soundings, released 10 miles apart in early 1969, the RMS error in determining geopotential heights at 300 mb varies from 56 ft (17m) for unchecked observations to 47 ft (14m) for observations which have been edited or screened. These figures are based on estimated temperature errors of 0.4°C and 0.3°C respectively, combined with a pressure error of 3 mb. Lenhard (1973) has recently revised his estimate down to 8.5 m for soundings subject to systematic consistency checks.

The above estimate is consistent with RMS random error of 8 m given in Table 1 for OWS Charley but is somewhat less than that of 13 m attributed to the US-type sonde at Keflavik. Both are considerably less than the 20-21 m RMS difference in geopotential heights at 300 mb between these two stations given in Table 2. The figures in Tables 1 and 2 were provided by A. H. Hooper, Chairman of the WMO Working Group on Radiosonde instruments and measurements, and are based on a series of comparisons made in 1973. The comparisons were made at 100 mb where the RMS radiosonde errors can be separated from the variability of the atmosphere. The errors thus found are multiplied by a factor of 0.45 to reduce them to their estimated value at 300 mb.

Because of the apparent discrepancy in the error estimates discussed above, we shall adopt two values for the RMS random errors of US-type radiosondes: an RMS random error of 10 m which is close to the lowest estimate available, and one of 20 m which is close to the highest.

Over the Atlantic where different types of sondes are used we shall also adopt two estimates for the quantity $(\overline{\varepsilon_1^2} + \overline{\varepsilon_2^2})$ in equation 8. The lower estimate is derived from the average of the 15 combinations possible from the six radiosondes listed in Table 1 and is equivalent to an RMS random error of 18.3 m; the higher estimate is based on the average of the RMS differences in Table 2 and is equivalent to an RMS random error of 21.3 m.

Table 1* RMS random geopotential errors at 300 mb associated with different radiosondes, estimated as 0.45 times the standard deviation of the reported geopotentials about their population mean at 100 mb.

Type of Radiosonde	Location	RMSE
US	Keflavik	13 m
US	OWS Charley	8 m
Vaisala	OWS Mike	19 m
French	OWS King	12 m
UK	OWS Juliet	18 m
Dutch	DeBilt	12 m

Table 2* Best estimate of RMS difference in meters of geopotential height at 300 mb between US sonde at Keflavik and sondes of types and at locations for May 1973.

Type and location of sonde	00 GMT	12 GMT
Vaisala - OWS Mike	27	24
French - OWS King	17	17
UK - OWS Juliet	21	22
US - OWS Charley	20	21
Dutch - DeBilt	24	20

*Communicated by Mr. A. H. Hooper, Chairman of Working Group on radiosonde instruments and measurements to Mr. V. D. Rockney, President of CIMO.

b. Errors of satellite observations

Several investigations are in progress, comparing radiosonde observations with those from satellites. The results of these investigations vary over a wide range owing to the different manner in which the different comparisons are set up.

Table 2 shows the standard deviation of the thickness differences between 1000 mb and 300 mb, obtained from radiosondes and satellites during March 1973. The values are based on comparisons between

Table 3* Standard deviation in meters of the thickness difference between 1000 mb and 300 mb obtained from radiosondes and satellites during March 1973.

Latitude	Standard deviation (meters)	Sample size
18-30	48.75	58
30-40	48.86	53
40-50	63.21	62
50-60	55.13	71
60-70	67.77	44

radiosonde and satellite observations which are within 1°deg Lat in space and 6 hours in time. They therefore incorporate some spurious errors due to the variability of the atmosphere. We shall adopt these values as representative of unedited satellite observations. In doing so, we are assuming that the height of the 1000 mb surface can be accurately determined.

For satellite observations which are edited or screened we shall adopt the value of 42 m suggested by Dr. G. P. Cressman** on the basis

*Communicated by K. Johnson, Upper Air Branch, NMC.
**Oral communication.

of direct comparisons which he made between radiosonde observations and observations from satellites, after the latter had passed the screening criteria established by NMC.

The random RMS difference between radiosonde and satellite observations incorporates the random errors of both systems. To estimate the RMS random errors of satellites alone, let R_r, R_s and R_{rs} denote respectively the RMS random errors of radiosondes, satellites and the combined errors of both. Since R_r and R_s are independent

$$R_{rs}^2 = R_r^2 + R_s^2 \tag{14}$$

and

$$R_s = (R_{rs}^2 - R_r^2)^{1/2} \tag{15}$$

4. Computations and Results

We have applied equations (9) to (13) to compute mean square geopotential gradients and the errors involved in their determination from the present configuration of radiosonde and satellite (VTPR) observations over the North Atlantic and conterminous United States. We have assumed that the average distance between radiosondes is 1000 km over the North Atlantic and 330 Km over the conterminous United States. Estimates for the average distance between retrieved VTPR observations vary between 400 Km (Jastrow and Halem, 1973), at best, to 500 Km or more for the day-to-day operational average.

We have used a value of 180 m and 100 m respectively for the standard deviation of the geopotential in winter and summer. For the autocorrelation coefficients μ (r), we have adopted the functions computed over European Russia by Tatarskaya (1965) from 90 situations separated by 3 days for both winter and summer. While these functions are expected to be somewhat different from those over our area of interest, they provide a sufficiently good approximation for our present purpose.

a. Mean square gradients

Table 4 shows computed values of the mean square gradient G which are not contaminated by the random errors of observations. The gradients are computed for both winter and summer over varying distances r corresponding to each of the observational grids assumed above.

Table 4. True mean square geopotential gradients (G) at 300 mb, computed for winter and summer over varying distances r. Units are m^2/km^2.

	r (km)			
	330	400	500	1000
Winter	5.36×10^{-2}	4.86×10^{-2}	4.67×10^{-2}	3.2×10^{-2}
Summer	2.02×10^{-2}	1.75×10^{-2}	1.52×10^{-2}	$.96 \times 10^{-2}$

We note that the values vary appreciably, from winter to summer and also with varying values of r. Therefore, comparison between the performance of radiosondes and satellites is meaningful only if made for the same computational scale and season.

b. Computational errors over the North Atlantic.

Tables 5 and 6 show the computational errors E and the noise signal ratio η of 300 mb geopotential gradients computed from raw radiosonde and satellite observations over distances characteristic of the average observational grids, assumed earlier, or their multiples. Thus in figure 5 which relates to the North Atlantic, the computations are over 400, 500 and 1000 km for satellite observations and over 1000 km only for radiosonde observation. The radiosonde errors assumed are 18.3 and 21.3 corresponding respectively to the average of all possible combinations in Table 1, and the mean of RMS difference in Table 2. For satellite observations we have adopted 60 m and 42 m as representing the highest and lowest limits of the random RMS differences between these observations and radiosonde observations. We have further given satellite observations the benefit of the doubt by assuming that the above RMS differences incorporate an RMS random radiosonde error of 20 m. The reduces the RMS random errors of satellite observations to 57.0 m and 37.0 m.

Table 5. Errors (E) in the mean square 300 mb geopotential gradient in m^2/km^2, computed from radiosonde and satellite (VTPR) observations representative of the North Atlantic. The noise-signal ratio η = E/G.

	Season	$\sqrt{\overline{\varepsilon^2}}$ (m)	r (km) 400 10^3E	η	500 10^3E	η	1000 10^3E	η
R/S	Winter	18.3					.67	.02
	Summer							.07
	Winter	21.3					9.	.028
	Summer							.094
VTPR	Winter	37.0	17.1	.35	11.0	.24	2.7	.09
	Summer			.98		.72		.28
	Winter	57.0	40.6	.86	26.0	.56	6.5	.20
	Summer			2.3		1.7		.68

Table 5 clearly shows that even with our probable underestimate of the errors of VTPR observations, gradients estimated from the "raw" observations are all but useless when computed over distances of the order of 400-500 km. Thus in summer, the errors of gradients computed over these distances may exceed twice the values of the gradients themselves! The situation in winter is somewhat better but still far from satisfactory. Comparing gradients computed over 1000 km, we find that season for season, the noise-signal ratio associated with the best satellite observations is about 3 times that associated with the worst radiosonde observations.

The above comparison may not be entirely fair since it does not reflect the advantage of the greater density of satellite observations over the ocean. It is well known that observations subject to random errors may be improved by smoothing and that the improvement is commensurate with the closeness of the observations.

Table 6 shows the errors of gradients computed from observations which have been smoothed by optimum combination with the 4 nearest observations (Alaka and Elvander, 1972). Strictly speaking, this violates the assumption made in the derivation of equation (4) that the observational errors are independent of each other. However, the weight which each of the 4 neighboring observations contributes to the smoothed observation is only a small fraction of the total weight, so that, in effect, any violation of the assumption of error independence is probably not very serious.

Table 6. Errors and noise signal ratios of 300 mb geopotential gradients computed from optimally smoothed radiosonde and VTPR observations.

	SEASON	$\sqrt{\overline{\varepsilon^2}}$	r (km) 400		500		1000	
			10^3E	η	10^3E	η	10^3E	η
RAWINSONDE 1000 km GRID	Winter	18.3					0.66	.021
	Summer						0.62	.065
	Winter	21.3					0.87	.027
	Summer						0.82	.086
VTPR 500 km GRID	Winter	37.0			7.34	.157	1.84	.058
	Summer				5.55	.365	1.59	.145
	Winter	57.0			13.2	.283	3.30	.103
	Summer				9.60	.634	2.41	.251
VTPR 400 km GRID	Winter	37.0	8.80	.181				
	Summer		7.20	.410				
	Winter	57.0	16.1	.331				
	Summer		12.9	.737				

Comparison of Tables 5 and 6 clearly shows the improvement of smoothed over raw observations. The improvement increases with decreasing grid length and with increasing RMS errors. Therefore, smoothing is more advantageous to VTPR observations which are less accurate but are closer together than radiosonde observations. Thus when the smoothing is made over a 400 km grid, the noise-signal ratio for VTPR observations decreases by a factor of 2-3. On the other hand, the smoothing of radiosonde observations, over a 1000 km grid hardly improves the noise signal ratio at all. Nevertheless the noise signal ratio from the most accurate VTPR observations, smoothed over a 500 km grid, is still larger than that from the least accurate radiosonde observations, smoothed over a 1000 km grid by a factor of 2 in winter and 1.5 in summer.

In the above analysis, we have not taken into consideration the fact that radiosonde stations make wind observations. This is an important factor since each wind observation gives two pieces of information while a height observation gives only one. Thus a network of height observations d km apart is equivalent to one with both height and wind observations $d\sqrt{3}$ km apart. On this basis, the network of radiosondes over the Atlantic which make both wind and height observations and which are about 1000 km apart, is equivalent to one making height observations only over a 600 km grid. This fact detracts from the advantage of the greater apparent density of satellite observations over the ocean, and should contribute to an even greater inbalance between the comparative performance of radiosondes and satellites.

c. Errors in computing mean square Laplacians.

Table 7 gives the noise-signal ratio η' for the mean-square Laplacian computed from smoothed radiosonde and satellite data. If the Laplacian is computed from a network of VTPR observations 400 km apart, η' varies from about .68 to 1.84. Much better results are obtainable if r is 1000 km but the smoothing is performed from a network of observations 500 apart. Even so, as can be seen from the last column of Table 7, η' associated with the most accurate VTPR observation is larger than that associated with the least accurate radiosonde observations by a factor of 2.9 in winter and 2.2 in summer.

Table 7. Noise signal ratio η' of Laplacians computed from smoothed radiosonde and VTPR observations over the North Atlantic.

	SEASON	$\sqrt{\overline{\varepsilon^2}}$ (m)	r (km)		
			400	500	1000
RADIOSONDE 1000km GRID	Winter	18.3			.036
	Summer				.092
RADIOSONDE 1000km GRID	Winter	21.3			.043
	Summer				.122
VTPR 500 km GRID	Winter	37.0		.417	.103
	Summer			.787	.206
	Winter	57.0		.749	.184
	Summer			1.37	.357
VTPR 400 km GRID	Winter	37.0	.680		
	Summer		1.03		
	Winter	57.0	1.23		
	Summer		1.84		

d. Computational errors over the conterminous U.S.

Over the conterminous U.S., comparison of the effectiveness of the current network of radiosondes with retrieved observations from satellites should be even less favorable to the latter. There are two main reasons for this: first, the retrieved observations overland are not expected to be nearly as accurate as those over the ocean; secondly, the radiosonde network is denser than the most optimistic estimate of the expected network of retrieved satellite observations, so that smoothing is no longer predominantly in favor of the latter.

In Tables 8 and 9, we have adopted two values for the RMS errors of U.S.-type radiosondes. These, as mentioned earlier, are close to the highest and lowest estimates available on these instruments. For satellites we have adopted a value of 56 m which is close to the average standard deviations from 30–50°L in Table 3. From this we extracted an RMS error of 20 m for radiosondes in accordance with equation (15). The result is and RMS error of 52.3 which we again consider to be an underestimate of the true errors.

Table 8. Errors (E) in the mean square 300 mb geopotential gradient in m^2/km^2, computed from raw radiosonde and satellite (VTPR) observations over the conterminous United States. The noise-signal ratio $\eta = \frac{E}{G}$.

	SEASON	$\sqrt{\overline{\varepsilon^2}}$ (m)	r (km) 330 10^3E	330 η	400 10^3E	400 η	500 10^3E	500 η
RAWINSONDE	Winter	20	7.35	.137	5.00	.103	3.20	.069
	Summer			.364		.286		.211
	Winter	10	1.84	.034	1.25	.026	0.80	.017
	Summer			.091		.071		.053
VTPR	Winter	52.3			34.2	.70	21.9	.47
	Summer					1.95		1.44

Table 9. Errors (E) and noise-signal ratios (η)of 300 mb gradients computed from optimally smoothed radiosonde and VTPR observations over the conterminous United States.

	SEASON	$\sqrt{\overline{\varepsilon^2}}$ (m)	r (km) 330 10^3E	330 η	400 10^3E	400 η	500 10^3E	500 η
RAWINSONDE 330 km GRID	Winter	20	5.63	.106	3.83	.079	2.45	.052
	Summer		4.01	.199	2.74	.157	1.75	.115
	Winter	10	1.71	.032	1.17	.024	0.75	.016
	Summer		1.46	.072	0.99	.057	0.64	.042
VTPR 500km GRID	Winter	52.3					11.8	.252
	Summer						8.63	.568
VTPR 400km GRID	Winter	52.3			13.8	.284		
	Summer				11.5	.657		

Table 8 which gives the results for raw observations shows that at best, the noise signal ratio η for satellite observations is 6.8 times that for radiosonde observations. This factor is reduced to about 5 for smoothed observations, as can be seen from Table 9.

5. References

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