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A Method for Predicting Surface Winds

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A METHOD FOR PREDICTING SURFACE WINDS

Harry R. Glahn

Systems Development Office
Techniques Development Laboratory

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CONTENTS

A METHOD FOR PREDICTING SURFACE WINDS

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ABSTRACT

A method is presented which can be used operationally in forecasting surface winds. Various regression models are discussed and applied to available data. Verification on independent data indicates that this method produces short range forecasts of wind of equal or greater accuracy than the official aviation terminal wind forecasts (FT's).

INTRODUCTION

One of the meteorological variables which is seldom forecast directly by numerical models is surface wind. Estimates of this variable may be obtained from certain experimental models or from the Primitive Equation (PE) Model [1] or Subsynoptic Advection Model (SAM) [2] being used by the Weather Bureau. Neither of these operational models takes into account the local topography which considerably affects the wind.

Probably the best way to arrive at an objective estimate of the surface wind is to statistically relate the observed wind to the forecasts of wind, and perhaps other variables, obtained from the numerical models. In order to use this Model Output Statistics (MOS) technique, a sample of the forecasts from the model must be collected for analysis. Such a sample of predictor data as well as predictand data was available from TDL's SAM Project [2].

This paper discusses statistical models which may be appropriate for surface wind estimation, and results of application are given. The appendix contains formulas for the determination of the standard errors and regression constants and coefficients.

DATA SAMPLE

The data sample available from the SAM Project [2] contained in part 0700, 1200, 1500, 1800, 2100, and 0000 GMT observed winds at 100 cities in the eastern U.S. and a large number of predictor variables from the SAM and PE models for 203 days from April to September in 1967 and 1968. The predictor variables included 1000-mb geostrophic wind, 1000- and 500-mb temperature, sea level pressure, saturation deficit, relative humidity, and precipitation amount. These data were used in studying models for making objective forecasts of wind as described below.

REGRESSION MODELS

Specifying a two-dimensional wind vector presents some interesting problems. A general linear model arises from determining the coefficients in the two equations

$$\hat{u} = \sum_{i=0}^{p} a_i x_i$$

GENERAL MODEL

$$\hat{v} = \sum_{i=0}^{q} b_i y_i$$

where $x_0 = y_0 \equiv 1$

such that over the developmental sample

$$\sum |\mathbb{v} - \hat{\mathbb{v}}|^2 = \sum | \mathbb{i}(u-\hat{u}) + \mathbb{j}(v-\hat{v})|^2$$

$$= \sum (u-\hat{u})^2 + \sum (v-\hat{v})^2$$

$$= \text{minimum}$$

When no other restrictions are imposed on the a_i and b_i, solution of the normal equations developed for each component separately gives the best least squares fit on the sample for the total vector.

The reduction of variance of the vector wind can be written

$$RV_{\mathbb{v}} = \frac{\sigma_{\mathbb{v}}^2 - SE_{\mathbb{v}}^2}{\sigma_{\mathbb{v}}^2}$$

where $\sigma_{\mathbb{v}}$ = sample standard deviation

and $SE_{\mathbb{v}}$ = standard error.

Since $\sigma_{\mathbb{v}}^2 = \sigma_u^2 + \sigma_v^2$

and $SE_{\mathbb{v}}^2 = SE_u^2 + SE_v^2 = \sigma_v^2(1-R_v^2) + \sigma_u^2(1-R_u^2)$

for this model,

$$RV_{\mathbb{v}} = \frac{R_u^2 \sigma_u^2 + R_v^2 \sigma_v^2}{\sigma_u^2 + \sigma_v^2}$$

where R_u = the multiple correlation of u with all of its predictors and

R_v = the multiple correlation of v with all of its predictors.

For some applications, it may be that about the only useful predictor for surface wind is some other wind vector, such as a boundary layer wind or 1000-mb geostrophic wind from a numerical model or the wind at the same location at a previous time. The general model then becomes

$$\hat{u} = a_0 + a_1 u_0 + a_2 v_0$$

MODEL 1

$$\hat{v} = b_0 + b_1 u_0 + b_2 v_0$$

In this formulation, the predicted wind $\mathbb{V}$ is the sum of a constant vector and another vector obtained by stretching and turning the predictor wind $\mathbb{V}_0$ by an amount which varies with $\mathbb{V}_0$ itself.

Court [3] refers to the square root of RV_w determined from Model 1 as the "vector correlation coefficient" of $\mathbb{V}$ on $\mathbb{V}_0$. In an analogous manner, where the predictors are the same for u and v in the General Model and are vector components, the square root of RV_w determined from that model can be called the "multiple vector correlation coefficient" of $\mathbb{V}$ on the predictor vectors.

An even more general regression model arises out of the canonical correlation technique in which one set of variables is linearly related to another set of variables. The resulting "composite correlation coefficient" $R_{y \cdot x}$ discussed by Glahn [5] can be determined in exactly the same way as the multiple vector correlation coefficient described above.

Model 1 was used to predict the surface wind at the four times: 1500, 1800, 2100, and 0000 GMT from the 1000-mb geostrophic wind predicted by SAM valid at the same time; the SAM forecasts were for projections of 8, 11, 14, and 17 hours respectively. The sample consisted of 72 days during April through October 1967. Because of the small sample size, the generalized operator concept was used where data from 100 stations were combined. The resulting equations and their implications are shown in Figs. 1 and 2.

Since these equations represent an "average" relationship at 100 stations, the effects of local topography are not accounted for, except as they contribute to an "average" frictional effect. These equations do not give the relationship between surface wind and true 1000-mb geostrophic wind but rather between surface wind and 1000-mb geostrophic wind as predicted by SAM. Therefore, the decrease in speed from $\mathbb{V}_0$ to $\mathbb{V}$ takes into account the prediction errors in $\mathbb{V}_0$.

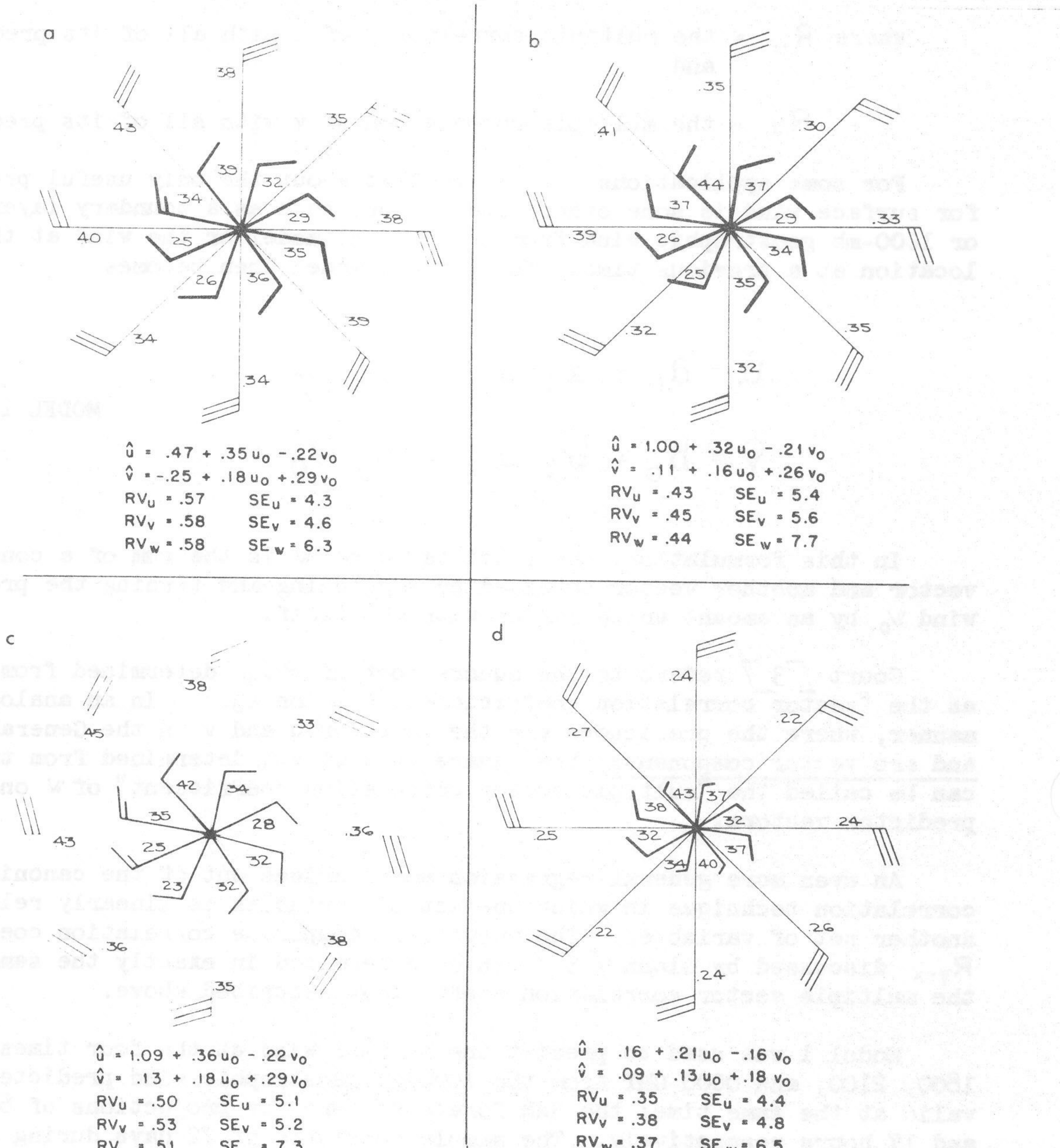


Figure 1.--Equations for estimating surface wind at 1500 GMT (a), 1800 GMT (b), 2100 GMT (c), and 0000 GMT (d), related statistics, and surface winds estimated from 30 kt SAM winds. The light-line winds from the eight points of the compass represent 30 kt 1000-mb geostrophic winds predicted by SAM. The value plotted along the geostrophic wind shaft is the ratio of the surface wind to the geostrophic wind. The value plotted between the geostrophic wind and the heavy-line wind representing the estimated surface wind is the number of degrees the surface wind is turned counterclockwise to this geostrophic wind. For instance, a west geostrophic wind of 30 kts would indicate a 245 degree surface wind of 12 kts at 1500 GMT.

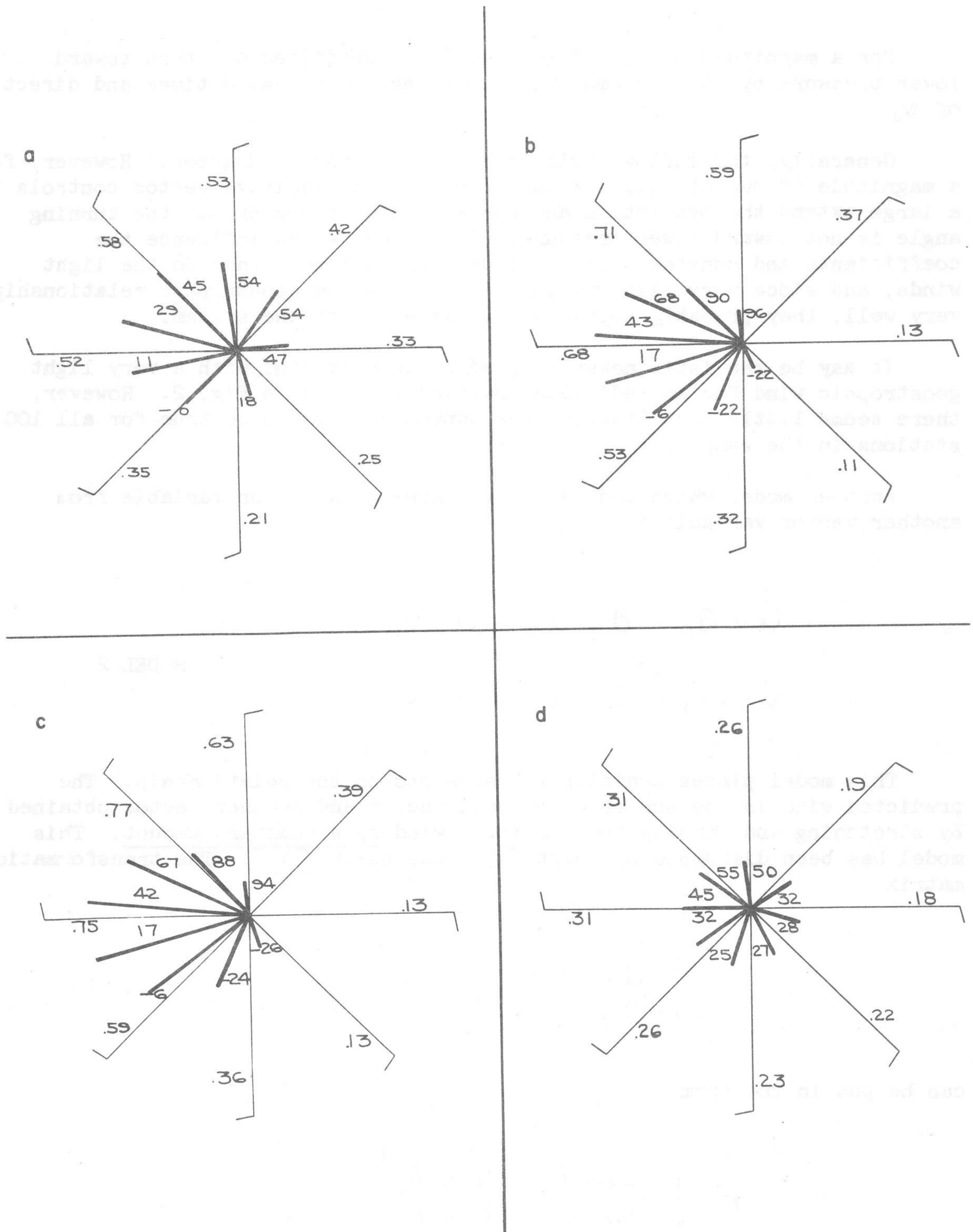


Figure 2.--Surface winds implied by 3 kt 1000-mb SAM winds at 1500 GMT (a), 1800 GMT (b), 2100 GMT (c), and 0000 GMT (d). The equations and plotting model are shown in Fig. 1.

For a magnitude of $\mathbb{V}_0$ of 30 knots, the predicted $\mathbb{V}$'s turn toward lower pressure by varying amounts, depending on the valid times and direction of $\mathbb{V}_0$.

Generally, the inflow angle is between 25 and 40 degrees. However, for a magnitude of $\mathbb{V}_0$ of only 3 knots, the constant additive vector controls to a large extent the prediction and for many directions of $\mathbb{V}_0$ the turning angle is not toward lower pressure. The strong winds influence the coefficients and constants in the equations much more than do the light winds, and since very light winds don't follow the geostrophic relationship very well, they probably appear mostly as noise to the system.

It may be realistic never to predict an east wind with a very light geostrophic wind for an individual station as shown in Fig. 2. However, there seems little justification for supposing this to be true for all 100 stations in the sample.

Another model which can be used to specify a vector variable from another vector variable is

$$\hat{u} = a_0 + a_1 u_0 + a_2 v_0$$
$$\hat{v} = b_0 - a_2 u_0 + a_1 v_0 \qquad \text{MODEL 2}$$

This model places additional restraints on the relationship. The predicted wind is the sum of a constant vector and another vector obtained by stretching and turning the predictor wind by a constant amount. This model has been discussed by Court [3] and Lewis [4]. The transformation matrix

$$\begin{vmatrix} a_1 & a_2 \\ -a_2 & a_1 \end{vmatrix}$$

can be put in the form

$$\beta \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$$

where β is the stretch ratio and θ is the turning angle.

Prediction equations for the 1800 GMT wind are indicated in Fig. 3.

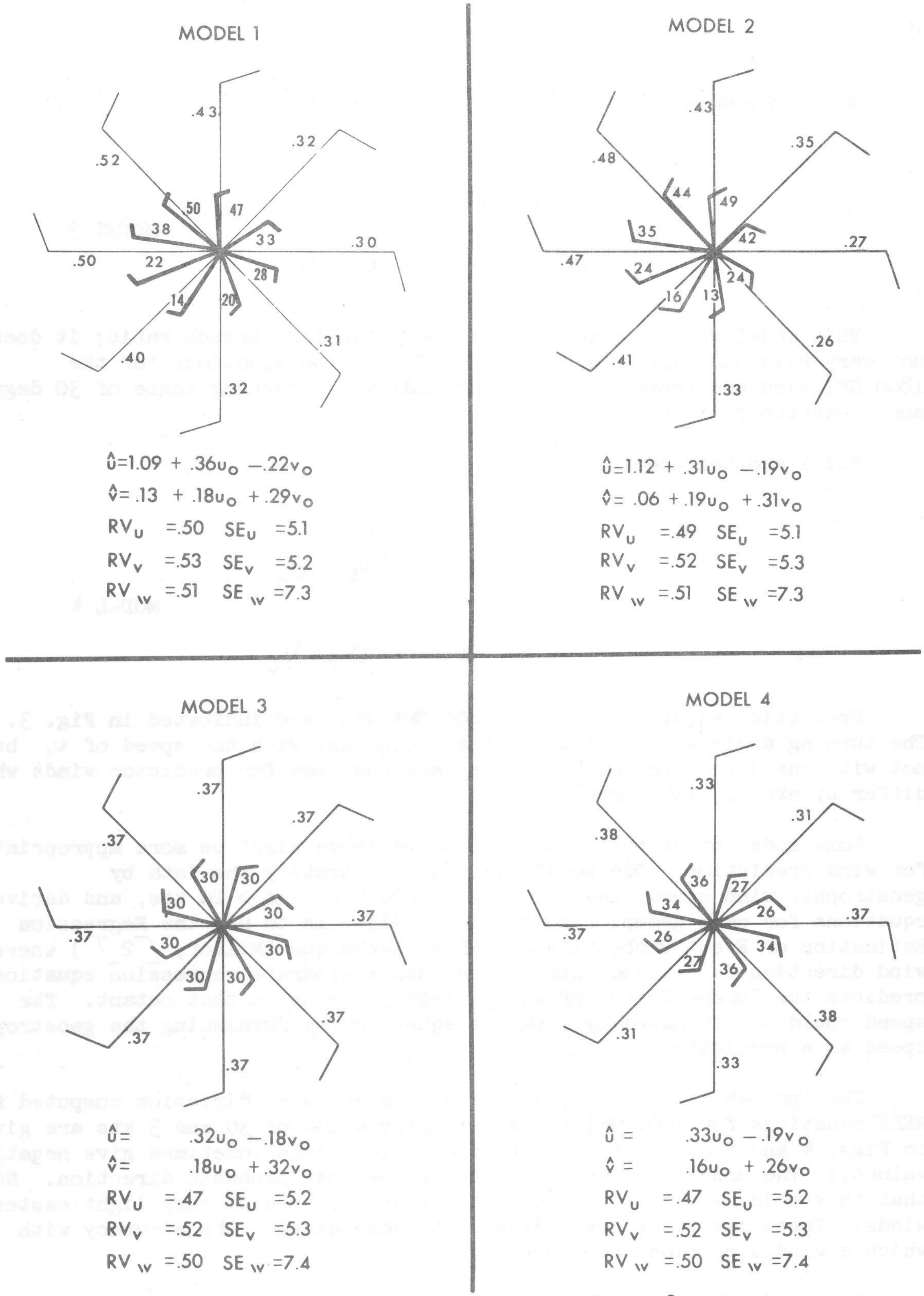


Figure 3.--Equations for estimating surface wind at 1800 GMT by four regression models and the resulting surface winds estimated from 10 kt 1000-mb geostrophic winds predicted by SAM. The plotting model is described in Fig. 1.

A third model is

$$\hat{u} = a_1 u_0 + a_2 v_0$$
$$\hat{v} = -a_2 u_0 + a_1 v_0 \qquad \text{MODEL 3}$$

This model gives a constant turning angle and stretch ratio; it does not vary with V_0 direction or speed. Prediction equations for the 1800 GMT wind are shown in Fig. 3 and indicate a turning angle of 30 degrees and a stretch ratio of .37.

Still another model is

$$\hat{u} = a_1 u_0 + a_2 v_0$$
$$\hat{v} = b_1 u_0 + b_2 v_0 \qquad \text{MODEL 4}$$

Prediction equations for the 1800 GMT wind are indicated in Fig. 3. The turning angle and stretch ratio are constant with the speed of V_0 but not with the direction; however, they are the same for predictor winds which differ by exactly 180 degrees.

Some model other than those discussed above might be more appropriate for wind prediction. One possibility is to stratify the data by geostrophic wind speed, say < 20 kts, 10-20 kts, and > 20 kts, and derive equations for each group. Another possibility is to use the Regression Estimation of Event Probabilities (REEP) technique (Miller, [2]) where the wind direction is divided into octants and a separate regression equation predicts the "probability" of wind direction being in that octant. The speed could be estimated by separate equations by furnishing the geostrophic speed as a predictor.

The "probability" distributions of surface wind direction computed from REEP equations for 1800 GMT for geostrophic winds of 30 and 3 kts are given in Figs. 4 and 5 respectively. (The REEP equations sometimes give negative values.) The underlined value indicates the most probable direction. Note that this model would not, like models 1 and 2, predict very light easterly winds. These distributions indicate, to some extent, the accuracy with which a wind direction can be predicted.

Regression equations which individually minimize the RMSE of the components u and v of a vector also minimize the mean square vector error. However, the same equations do not necessarily minimize the mean square error of the magnitude of the vector.

GEOSTROPHIC WIND DIRECTION	SURFACE WIND DIRECTION N	NE	E	SE	S	SW	W	NW
W	.01	-.09	-.10	-.01	.23	<u>.40</u>	.37	.19
SW	-.05	-.03	.03	.13	<u>.40</u>	.34	.17	.01
S	.01	.11	.20	.25	<u>.42</u>	.17	-.05	-.11
SE	.15	.24	<u>.31</u>	.28	.29	-.02	-.16	-.09
E	<u>.30</u>	.29	<u>.30</u>	.20	.08	-.12	-.10	.05
NE	<u>.35</u>	.24	.17	.05	-.08	-.06	.10	.23
N	.29	.10	.01	-.07	-.11	.11	.32	<u>.35</u>
NW	.15	-.04	-.11	-.10	.02	.30	<u>.44</u>	.34

GEOSTROPHIC WIND SPEED = 30 KTS.

Figure 4.--"Probability" distribution of surface wind direction for 30 kt 1000-mb geostrophic winds from SAM computed from the REEP equations. The most probable surface wind direction for each 1000-mb wind direction is indicated by underlining.

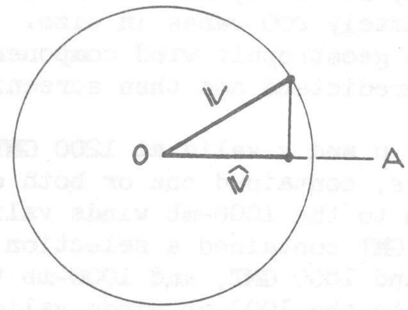
GEOSTROPHIC WIND DIRECTION	SURFACE WIND DIRECTION							
	N	NE	E	SE	S	SW	W	NW
W	.13	.08	.08	.08	.16	$\underline{.17}$	.16	.13
SW	.13	.09	.10	.09	$\underline{.18}$	.16	.14	.11
S	.14	.10	.11	.11	$\underline{.18}$	.14	.12	.10
SE	.15	.12	.13	.11	$\underline{.17}$	.12	.10	.10
E	$\underline{.17}$	.12	.12	.10	.15	.11	.11	.12
NE	$\underline{.17}$	.12	.11	.09	.13	.12	.13	.13
N	$\underline{.16}$	.10	.09	.08	.13	.14	.15	.15
NW	.15	.09	.08	.07	.14	.16	$\underline{.17}$	.14

GEOSTROPHIC WIND SPEED = 3 KTS.

Figure 5.--"Probability" distribution of surface wind direction for 3 kt 1000-mb geostrophic winds from SAM computed from the REEP equations. The most probable surface wind direction for each 1000-mb wind direction is indicated by underlining.

In order to illustrate this, let us consider winds which are always east or west, that is, $v = 0$. Also assume that $\bar{u} = 0$. Regression estimates $\hat{u}$ of u made on dependent data give $\bar{\hat{u}} = 0$ and $\sigma_{\hat{u}}^2 = R^2 \sigma_u^2$. However $\overline{|u|} \neq 0$ and $\overline{|\hat{u}|}$ will be less than $\overline{|u|}$ for observed distributions of u.

Another way to see this qualitatively is to consider the observed vector $\mathbb{V}$ in the figure below.



Now, $\hat{\mathbb{V}}$ will not, in general, be of exactly the correct direction, and may lie along line OA. If the square of the vector error is minimized in this individual case, the magnitude of $\hat{\mathbb{V}}$ would be such that $\mathbb{V} - \hat{\mathbb{V}}$ would be perpendicular to $\hat{\mathbb{V}}$ as indicated in the figure. Therefore $|\hat{\mathbb{V}}| < |\mathbb{V}|$ in the mean and will be a biased forecast of the wind speed.

(It is interesting to note that the mean direction error should not, in general, be $> 90°$ since this would imply a larger vector error than an estimate of $\hat{\mathbb{V}} = 0$.)

Unbiased estimates of wind speed can be obtained from a regression equation which estimates the speed directly. In this case, the predictors should include the speed of related vectors and not just their individual components. If it is desired to minimize the mean square error of the wind speed estimates, this procedure should be used rather than that of obtaining separate equations for the components.

Minimizing the mean square error of the individual component estimates does not minimize the mean square error of the direction computed from those estimates. Regression estimation of wind direction directly poses a special problem because of the circular nature of the variable. Possibilities exist that if the predictors include a vector that is rather well related to the predictand, the direction difference between that vector and the predictand can be used to define a new predictand with a scale of -180 to +180. The same basic problem exists with this new predictand as with the original with a scale of 0 to 360. However, the new predictand may usually lie in the range -90 to +90 and if so, perhaps omitting the few truant cases will produce a good result.

INDEPENDENT DATA VERIFICATION

Separate regression equations were developed for estimating the u and v wind components and the wind speed valid at 1200 and 1800 GMT for each of 10 stations in the eastern U.S. (The equations for the u and v components correspond to the General Model discussed earlier.) Data were used from the SAM project sample described above for the periods April through September 1967 and 1968. The stations were Albany, Atlanta, Baltimore, Cleveland, Cincinnati, Washington, New York, New Orleans, Chicago, and St. Louis. Each sample was of approximately 200 cases in size. The equations were developed by forcing the 1000-mb geostrophic wind components forecast by SAM valid at the same time as the predictand and then screening several other variables.

The equations for u and v valid at 1200 GMT selected for testing usually, but not always, contained one or both of the observed 0700 GMT wind components in addition to the 1000-mb winds valid at 1200 GMT; those used for testing valid at 1800 GMT contained a selection of 1000-mb winds at 1200 GMT, 500-mb winds at 1200 and 1800 GMT, and 1000-mb temperature at 1200 and 1800 GMT, in addition to the 1000-mb winds valid at 1800 GMT. The 1200 GMT equations contained from two to four predictors; the 1800 GMT equations contained from two to five predictors. The decision of which equation to test (how many predictors to include) was made subjectively.

The equations for estimating speed directly were derived by forcing the 1000-mb geostrophic wind speed forecast by SAM valid at the same time as the predictand and then screening several other variables. The equations used for testing contained from four to six predictors similar to those in the u and v equations, the only significant difference being that wind speeds were used as predictors whereas in the u and v equations only wind components were used as predictors.

Sample equations for St. Louis are shown below:

$$\hat{u}^{12} = .482 + .185\, u_0^{12} - .333\, v_0^{12} + .276\, v^{07}$$

$$\hat{v}^{12} = .194 + .164\, u_0^{12} + .175\, v_0^{12} - .005\, u^{07} + .170\, v^{07}$$

$$\hat{s}^{12} = 1.576 + .239\, s_0^{12} + .175\, s^{07} - .040\, v_0^{18} + .027\, u_0^{12}$$

where u, v, and s are the u-wind component, v-wind component, and wind speed respectively in kts; the subscript 0 indicates 1000-mb geostrophic values in kts predicted by SAM; and the superscript indicates the valid time in GMT.

Thus, the estimate of wind speed at 1200 GMT (s^{12}) at St. Louis depends on the 1200 GMT 1000-mb geostrophic wind speed forecast by SAM (s_0^{12}), the observed 0700 GMT surface wind speed at St. Louis (s^{07}), the 1800 GMT 1000-mb v-wind component forecast by SAM (v_0^{18}), and 1200 GMT 1000-mb u-wind component forecast by SAM (u_0^{12}).

The equations were evaluated for each day in April and May 1969 for which SAM data tapes were available. The wind forecasts in the FT's made at the Weather Bureau offices were used for comparison. Since the FT's do not mention wind if the speed is expected to be less than 10 kts, the comparison was made in two ways.

For all those cases where the FT's included wind and objective forecasts were available, the root mean square error (RMSE) of direction (computed from the u and v equations) and speed (direct from the speed equation) and the bias (mean forecast minus mean observed) of speed (both direct from the speed equation and calculated from the u and v equations) were computed. Also, for all cases when the FT's and objective forecasts were available, contingency tables for speed were prepared by considering the FT forecast of wind to be under 10 kts when wind was not mentioned. From these contingency tables, which had categories < 10, 10-12, 13-17, 18-22, and > 22 kts, skill scores and percent correct were computed. These scores are shown in Table 1.

VALID TIME (GMT)	PROJECTION (HR)	FORECAST	DIRECTION RMSE (DEG)	SPEED (kts)					
				RMSE	SKILL SCORE	PERCENT CORRECT	MEAN FORECAST	MEAN OBSERVED	BIAS
12	5	OBJECTIVE U,V EQUATIONS	166 35				167 9.4	167	-1.0
	5	OBJECTIVE SPEED EQUATION		167 3.5	539 .37	539 76	167 9.8	10.4	-.6
	3	FT	166 33	167 3.6	539 .36	539 71	167 12.0		1.6
18	11	OBJECTIVE U,V EQUATIONS	330 47				328 9.2	328	-1.9
	11	OBJECTIVE SPEED EQUATION		328 3.5	545 .29	545 54	328 11.3	11.1	.2
	9	FT	330 50	328 4.3	545 .24	545 49	328 12.6		1.5

Table 1. Comparison of official FT and objective wind forecasts for 10 stations in the eastern U.S. for April and May 1969. The number of cases used in calculating the statistic is in the upper right corner of the respective box.

Table 1 indicates that the directions from the objective forecasts were as good as those from the FT's and that the speeds from the objective were better than those from the FT's. The projections of the objective forecasts (5 and 11 hours) refer to the latest data used (0700 GMT). Actually the forecasts could be available to the field forecasters before 0900 GMT. The FT's were prepared with 0900 and perhaps 1000 GMT data available; transmission time for the forecasts is 1045 GMT. The bias in the speed computed from the u and v equations is noticeable.

CONCLUSION

Although the verification on independent data presented here is not prodigious, I believe it is sufficient to demonstrate the usefulness of this objective technique for surface wind forecasting. Further evaluation may prove that the objective forecasts can be issued directly to the user in most routine situations.

ACKNOWLEDGMENTS

This study was possible only through the use of the data collected in the SAM project. Many people participated in the collection and preparation of these data, particularly Mr. Dale Lowry, Mr. George Hollenbaugh, Mrs. Jackie Hughes, Mrs. Evelyn Boston, Mr. Harry Akens, and Mrs. Elizabeth Booth. Mr. Roger Allen, Mrs. Mary Abernathy, and Mr. Harold Cole assisted with the verification; Miss Loretta Thompson and Mr. Herman Perrotti prepared the manuscript for printing. A portion of the work was supported by the Federal Aviation Administration.

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APPENDIX

Formulas for the evaluation of the constants and coefficients in the linear regression models which relate one two-dimensional vector $\mathbb{V}_0$ to another two-dimensional vector $\mathbb{V}$ are given below. The general formula for computing the standard error is:

$$
\begin{aligned}
SE^2 = {} & \sigma_u^2 + \bar{u}^2 - 2a_0\bar{u} - 2a_1(\sigma_{uu_0} + \bar{u}\bar{u}_0) - 2a_2(\sigma_{uv_0} + \bar{u}\bar{v}_0) \\
& + a_0^2 + 2a_0a_1\bar{u}_0 + 2a_0a_2\bar{v}_0 + a_1^2(\sigma_{u_0}^2 + \bar{u}_0^2) \\
& + 2a_1a_2(\sigma_{u_0v_0} + \bar{u}_0\bar{v}_0) + a_2^2(\sigma_{v_0}^2 + \bar{v}_0^2) \\
& + \sigma_v^2 + \bar{v}^2 - 2b_0\bar{v} - 2b_1(\sigma_{vu_0} + \bar{v}\bar{u}_0) - 2b_2(\sigma_{vv_0} + \bar{v}\bar{v}_0) \\
& + b_0^2 + 2b_0b_1\bar{u}_0 + 2b_0b_2\bar{v}_0 + b_1^2(\sigma_{u_0}^2 + \bar{u}_0^2) \\
& + 2b_1b_2(\sigma_{u_0v_0} + \bar{u}_0\bar{v}_0) + b_2^2(\sigma_{v_0}^2 + \bar{v}_0^2)
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{u} &= a_0 + a_1u_0 + a_2v_0 \\
\hat{v} &= b_0 + b_1u_0 + b_2v_0 \\
\sigma_u^2 &= \tfrac{1}{n}\Sigma u^2 - \bar{u}^2 = \text{sample variance of u, etc.} \\
\sigma_{uv} &= \tfrac{1}{n}\Sigma uv - \bar{u}\bar{v} = \text{sample covariance of u and v, etc.} \\
\bar{u} &= \text{mean of u, etc.}
\end{aligned}
$$

This formula can be used for Models 1 through 4. For Model 1 a much simpler relationship is given in the text. For each of the other models, some simplification is possible because some constants will be zero or certain coefficients are equal.

MODEL 1

$$
\begin{aligned}
\hat{u} &= a_0 + a_1u_0 + a_2v_0 \\
\hat{v} &= b_0 + b_1u_0 + b_2v_0
\end{aligned}
$$

$$a_1 = \frac{\sigma_{uu_0}\,\sigma_{v_0}^2 - \sigma_{uv_0}\,\sigma_{u_0v_0}}{\sigma_{u_0}^2\,\sigma_{v_0}^2 - \sigma_{u_0v_0}^2}$$

$$a_2 = \frac{\sigma_{uv_0}\,\sigma_{u_0}^2 - \sigma_{uu_0}\,\sigma_{u_0v_0}}{\sigma_{u_0}^2\,\sigma_{v_0}^2 - \sigma_{u_0v_0}^2}$$

$$a_0 = \bar{u} - a_1\bar{u}_0 - a_2\bar{v}_0$$

$$b_1 = \frac{\sigma_{vu_o}\sigma_{v_o}^2 - \sigma_{vv_o}\sigma_{u_o v_o}}{\sigma_{u_o}^2\sigma_{v_o}^2 - \sigma_{u_o v_o}^2}$$

$$b_2 = \frac{\sigma_{vv_o}\sigma_{u_o}^2 - \sigma_{vv_o}\sigma_{u_o v_o}}{\sigma_{u_o}^2\sigma_{v_o}^2 - \sigma_{u_o v_o}^2}$$

$$b_o = \bar{v} - b_1\bar{u}_o - b_2\bar{v}_o$$

a_0, a_1, and a_2 are not dependent on v, and b_0, b_1, and b_2 are not dependent on u.

MODEL 2

$$\hat{u} = a_o + a_1 u_o + a_2 v_o$$

$$\hat{v} = b_o - a_2 u_o + a_1 v_o$$

$$a_1 = \frac{\sigma_{uu_o} + \sigma_{vv_o}}{\sigma_{u_o}^2 + \sigma_{v_o}^2}$$

$$a_2 = \frac{\sigma_{uv_o} - \sigma_{vu_o}}{\sigma_{u_o}^2 + \sigma_{v_o}^2}$$

$$a_o = \bar{u} - a_1\bar{u}_o - a_2\bar{v}_o$$

$$b_o = \bar{v} + a_2\bar{u}_o - a_1\bar{v}_o$$

Each of the coefficients a_1 and a_2 and constants a_0 and b_0 are dependent on both u and v.

MODEL 3

$$\hat{u} = a_1 u_o + a_2 v_o$$

$$\hat{v} = -a_2 u_o + a_1 v_o$$

$$a_1 = \frac{\sigma_{uu_o} + \bar{u}\bar{u}_o + \sigma_{vv_o} + \bar{v}\bar{v}_o}{\sigma_{u_o}^2 + \bar{u}_o^2 + \sigma_{v_o}^2 + \bar{v}_o^2}$$

$$a_2 = \frac{\sigma_{uv_o} + \bar{u}\bar{v}_o - \sigma_{vu_o} - \bar{v}\bar{u}_o}{\sigma_{u_o}^2 + \bar{u}_o^2 + \sigma_{v_o}^2 + \bar{v}_o^2}$$

Each of the coefficients is dependent on both u and v.

MODEL 4

$$\hat{u} = a_1 u_0 + a_2 v_0$$

$$\hat{v} = b_1 u_0 + b_2 v_0$$

$$a_1 = \frac{(\sigma_{uu_0} + \bar{u}\bar{u}_0)(\sigma_{v_0}^2 + \bar{v}_0^2) - (\sigma_{uv_0} + \bar{u}\bar{v}_0)(\sigma_{u_0v_0} + \bar{u}_0\bar{v}_0)}{(\sigma_{u_0}^2 + \bar{u}_0^2)(\sigma_{v_0}^2 + \bar{v}_0^2) - (\sigma_{u_0v_0} + \bar{u}_0\bar{v}_0)^2}$$

$$a_2 = \frac{(\sigma_{uv_0} + \bar{u}\bar{v}_0)(\sigma_{u_0}^2 + \bar{u}_0^2) - (\sigma_{uu_0} + \bar{u}\bar{u}_0)(\sigma_{u_0v_0} + \bar{u}_0\bar{v}_0)}{(\sigma_{u_0}^2 + \bar{u}_0^2)(\sigma_{v_0}^2 + \bar{v}_0^2) - (\sigma_{u_0v_0} + \bar{u}_0\bar{v}_0)^2}$$

$$b_1 = \frac{(\sigma_{vu_0} + \bar{v}\bar{u}_0)(\sigma_{v_0}^2 + \bar{v}_0^2) - (\sigma_{vv_0} + \bar{v}\bar{v}_0)(\sigma_{u_0v_0} + \bar{u}_0\bar{v}_0)}{(\sigma_{u_0}^2 + \bar{u}_0^2)(\sigma_{v_0}^2 + \bar{v}_0^2) - (\sigma_{u_0v_0} + \bar{u}_0\bar{v}_0)^2}$$

$$b_2 = \frac{(\sigma_{vv_0} + \bar{v}\bar{v}_0)(\sigma_{u_0}^2 + \bar{u}_0^2) - (\sigma_{vu_0} + \bar{v}\bar{u}_0)(\sigma_{u_0v_0} + \bar{u}_0\bar{v}_0)}{(\sigma_{u_0}^2 + \bar{u}_0^2)(\sigma_{v_0}^2 + \bar{v}_0^2) - (\sigma_{u_0v_0} + \bar{u}_0\bar{v}_0)^2}$$

For this model a_1 and a_2 are independent of v, and b_1 and b_2 are independent of u.

(Continued from inside front cover)

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