Science and Technology Infusion Climate Bulletin
NOAA's National Weather Service
36th NOAA Annual Climate Diagnostics and Prediction Workshop
Fort Worth, TX, 3-6 October 2011

A Method for Monitoring Meteorological Drought at Daily Scale

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1. Introduction

Drought is difficult to monitor, and various indexes have been proposed to detect the drought as well as flood (*e.g.*, Dracup *et al.* 1980, Wilhite and Glantz 1985, Keyantash and Dracup 2002, Redmond 2002, Svoboda *et al.* 2002). The index previously widely used in the United States is the Palmer Drought Severity Index (PDSI) (*e.g.*, Palmer 1965, Karl and Knight 1985, Heddinghaus and Sabol 1991), which is based on the anomalies of the supply and demand components in the water balance equation. In addition to precipitation, the PDSI also needs to use temperature and other local hydrological quantities. It has been found that the PDSI has many significant limitations (*e.g.*, Alley 1984, Karl and Knight 1985, Smith *et al.* 1993, Willeke *et al.* 1994, Kogan 1995, McKee *et al.* 1995, Guttman 1998) and is not satisfied in the operational monitoring (*e.g.*, Hayes *et al.* 1999). Mo and Chelliah (2006) made some modifications to improve the PDSI.

The index currently widely used is the Standardized Precipitation Index (SPI) (McKee *et al.* 1993; McKee *et al.* 1995), which measures meteorological flood and drought and uses only precipitation. It compares the precipitation of a year with historical records and uses probabilities to indicate if the precipitation of the year is greater or less than the median precipitation of the historical records. Using the SPI, Hayes *et al.* (1999) examined the 1996 drought in the southern plains and southwestern United States. The SPI can determine whether a specific year of an area is a flood or drought year when compared with historical records, but cannot identify if this is a flood or drought area in space.

The major limitation of the SPI is that it can only be used to determine the flood and drought at scales of a month and longer. The SPI computed by the National Climate Data Center has scales ranging from one month to 24 months. Within the period of a scale, it treats the precipitation days equally and uses the simple average of the precipitation to indicate the general flood and drought situation of the period. The SPI does not take into account the precipitation before the period, hence cannot be used to measure flood and drought at shorter timescales (*e.g.*, less than a week). When this index is used to determine the flood and drought extent of the recent three days, for example, if there is no precipitation in the three days, then these three days may be regarded as a drought period when compared with historical records. However, if there was a very strong precipitation just before this period, then the recent three days may still be in a flood state.

Why should flood and drought be measured over the period of a scale, and the days of the period be treated equally? Although the general flood and drought situations of long periods (*e.g.*, several months or years) are useful for long-term planning, decision-makers of agriculture and other water-related departments also hope to know the present day flood and drought situation of a place or area, as well as the flood and drought tendency in the coming days (based on the prediction of precipitation). An index is thus required to measure the daily flood and drought extent. With the daily index, the start, duration, breaks, and strength of flood and drought can be determined. The interannual variabilities and long-term changes of these timings and the strength can further be evaluated. In reality, the flood and drought extent is a hydro-meteorological state of a land-atmosphere system and thus should possess a value at every moment. Once the daily values of the flood and drought extent are determined, the means of a month and longer periods can be calculated.

The purpose of this note is to provide physical considerations for developing such an index to measure the daily flood and drought extent from precipitation with the aim to determine the timings and strength of flood and drought. The rationale is that the flood and drought extent of a day depends on both the precipitation of

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the day and the precipitation of the previous days, but the influences from the precipitation of the previous days are decayed, which is due to the “demands” of the water balance, including the surface runoff, evapotranspiration, groundwater flow, and percolation.

2. Physical considerations of the approach

a. Theoretical framework

The variation process of flood and drought extent can be understood as in the way that the flood extent of a day is achieved through superposing the present day precipitation on the basis of the flood extent of the previous day. If there is large precipitation in the present day, flood extent may increase. If the large precipitation is persistent and lasts for several days, the flood extent will keep increasing, though gradually, and reach large values. Then in the dry days after the persistent precipitation, the flood extent will go down gradually to smaller values. This is due to the inherent decay mechanism of the soil-land surface system, which is caused by the demands of the water balance. Among the demands, runoff can be important during or just after the precipitation days, but in the later dry days evapotranspiration can be important.

Based on the above understanding, a simple physical model can be used to describe the variation of the flood extent. It uses only precipitation, but the general effect of the demands of water balance is considered. The model can be expressed as

$$\frac{df(t)}{dt} = -bf(t) + P(t), \tag{1}$$

where t is the time with the present moment being 0, and $f(t)$ the flood extent at time t in a location with its change being forced by precipitation $P(t)$. The total effect of the decay of flood extent from the demands of runoff, evapotranspiration, groundwater flow, and percolation is represented with $-b\,f(t)$, in which $b > 0$ measures the strength of the decay.

Integrating equation (1) with t from $-\infty$ to 0 yields

$$\left[e^{bt} f(t)\right]_{-\infty}^{0} = \int_{-\infty}^{0} e^{bt} P(t) dt, \tag{2}$$

or

$$f(0) = \int_{-\infty}^{0} e^{bt} P(t) dt, \tag{3}$$

which indicates that the flood extent at the present moment depends on the precipitation of the moment and all the earlier time, but the influence of the earlier time precipitation is reduced.

To use daily precipitation data, denote $t = -n\Delta t$, where n is the number of the day prior to the present day, and Δt = 1 day. Then (3) can be written as

$$f_0 = \Delta t \sum_{n=0}^{\infty} a^n P_n, \tag{4}$$

where $a = e^{-b\Delta t} < 1$ represents the strength of the contribution of the previous day precipitation P_1 to the present day flood extent f_0.

For practical use, (4) can be truncated as

$$f_0 = \Delta t \sum_{n=0}^{N} a^n P_n, \tag{5}$$

where N is the number of the earlier days used in the calculation, and can be determined from the parameter a and the c, a value much smaller than 1 (*e.g.*, 1%), indicating the precision of the truncation (the contribution fraction of the last day precipitation). With $a^N = c$, N can be determined as $N = \ln c / \ln a$.

The index being proposed for measuring the daily flood and drought extent can be in two forms. One is the *cumulation of reduced precipitation* (CRP), being defined from (5) as

$$CRP \equiv \sum_{n=0}^{N} a^n P_n \,, \tag{6}$$

which means that the flood extent of a day is contributed by the precipitation of the day on the basis of the reduced precipitation of the earlier days. The contribution strength of the earlier day precipitation relies on the parameter a. The CRP measures the absolute flood and drought extent, and can be used to study the spatial variability of the flood and drought extent.

The other form of the index is the *weighted average of precipitation* (WAP), which is defined as

$$WAP \equiv \sum_{n=0}^{N} a^n P_n \Big/ \sum_{n=0}^{N} a^n \,. \tag{7}$$

The SPI and the present day precipitation can be regarded as the two extreme cases of the general form of the WAP (in which a is between 0 and 1, and N can vary). When the contribution parameter a tends to be zero, the present day flood extent WAP will become the present day precipitation P_0. When a tends to be 1, WAP will be equivalent to the simple average of the precipitation of the present day and some earlier days as used in the SPI. The different length of the period $N+1$ (e.g., one or several months), over which the average is made, represents the different timescales of the SPI.

The WAP in (7) can further be written as

$$WAP = \sum_{n=0}^{N} w_n P_n \,, \tag{8}$$

where the weight $w_n = (1-a)a^n / (1-a^{N+1})$ can be simplified as

$$w_n = (1-a)a^n \,, \tag{9}$$

since $a^{N+1} = ac << 1$. The WAP measures the relative flood and drought extent of a specific location or area, and can be used to study the temporal variability of the flood and drought extent. For seasonal variations of a place, the start, duration, and strength of flood and drought can be determined. The interannual variabilities and long-term changes of the timings and strength can then be calculated.

b. Examples of using the WAP index

Since the major issue of flood and drought extent is the evaluation of its temporal variability at a place (*e.g.*, the start, duration, and strength of flood and drought and their interannual variations), examples of using the WAP index are provided here. Figure 1 presents an ideal time series of daily precipitation *P*, and shows how the calculated daily flood index WAP responds under different values of contribution parameter *a*. There are two episodes of large precipitation (10mm/day) in the 65-day period, and the remaining days in the period and the earlier days before day 1 all have small precipitation (1mm/day). In the first a few days after day 6, although precipitation has become large, the index WAP increases gradually from its

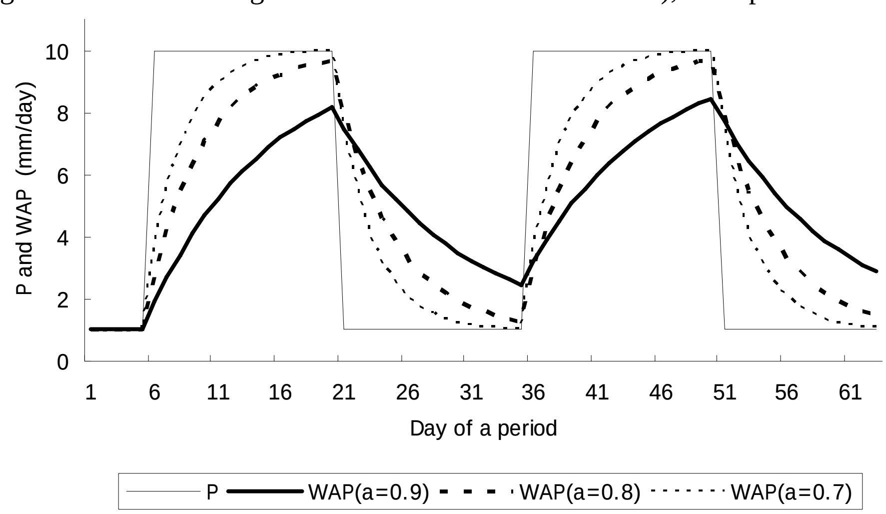

Fig. 1 An ideal time series of daily precipitation P and the calculated index WAP with the contribution parameter a being 0.9, 0.8, and 0.7.

small value of 1mm/day because of the small precipitation in the earlier days. The increase of WAP is relatively slow with *a* being 0.9. For smaller *a*, the increase of WAP is more rapid, so its curve is closer to that of *P*. As mentioned above, when *a* tends to be zero, the curve of WAP will tend to be the same as that of precipitation. With the persistent large precipitation, WAP increases steadily and becomes much larger at day 20. Similarly, WAP decreases gradually in the first a few days after day 21, when precipitation has become small. After a persistent small precipitation till day 35, WAP becomes very small. The decrease of WAP is also more rapid for smaller *a*. So, with the smoothing effect of the index, flood (large WAP) can occur after persistent strong precipitation, and drought (small WAP) may appear after persistent small or no precipitation.

Figure 2 shows an example of analyzing the seasonal and interannual variations of the flood and drought extent by use of the index WAP. The area studied is the upper Mississippi River basin that covers part of Minnesota, Wisconsin, Iowa, Illinois, and Missouri (*Lu et al.* 2009). The data, distributed by the National Climatic Data Center (www.ncdc.com), include the daily precipitation of over 500 observation stations in the basin from 1979 through 2004. A simple average of the daily precipitation is made over the stations of the basin. With taking *N* as 365, daily values of the index WAP from 1980 to 2004 are obtained. The seasonal cycles of the index in 1988 and 1993, when there were severe drought and flood respectively in the basin (*Glantz* 1988; *Changnon* 1996), and the averaged seasonal cycle of the 25 years are illustrated in Figure 2 with the parameter *a* being 0.9 and 0.8, respectively. It shows clearly the contrast of the flood and drought extent between the warm seasons of the two years. The different values of the parameter do not influence much the detections of the flood and drought. The values of the index in most of the days in the warm season of 1993 (1988) are greater (less) than the climatic values. The advantage of this index is that, compared with the SPI, the starting date and the duration of the flood and drought as well as the breaks in the duration can be determined with certain criteria. The comparison of the seasonal cycles of the two years with the climatic seasonal cycle indicates that, roughly, the flood of 1993 and the drought of 1988 both start from the first half of April. The flood of 1993 lasts until the end of September, and the drought of 1988 maintains for longer time. The breaks of the flood and drought can also be captured.

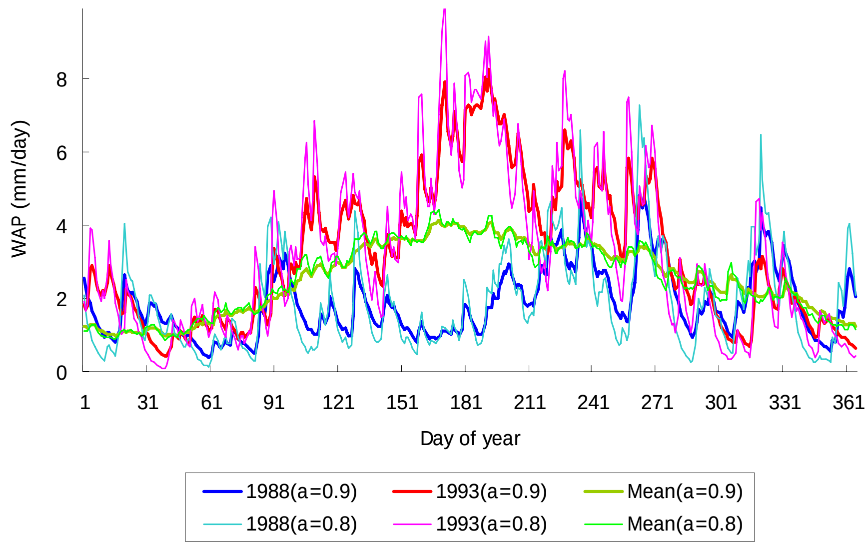

Fig. 2 Seasonal cycles of the daily index WAP in 1988, 1993, and the year averaged over 1980-2004 calculated from the observed daily precipitation averaged over the upper Mississippi River basin with the contribution parameter *a* being 0.9 and 0.8.

c. Contribution and weight parameters

The spatial variability of flood and drought extent can be examined with the index CRP, which measures the absolute flood and drought extent, and its value is dependent on the choice of the contribution parameter *a*. For meteorological flood and drought, which are caused by precipitation and consider the general characteristics of soil and land surface (*i.e.*, the general decay effect of the runoff, evapotranspiration, and percolation), an identical parameter can be used for different locations. Different values of *a* (*e.g.*, 0.9, 0.8, or 0.7) can be used respectively to evaluate the flood and drought extent, and this is somewhat like the different scales used in the SPI. If the specific characteristics of soil and land surface of each place need to be considered, then different values of *a* can be used for different places. The *a* can be chosen empirically since, after all, the flood and drought extent is what needs to be quantified, and there is no existing data of daily flood and drought extent to fit the parameter. It can be chosen by determining, based on the knowledge of the memory decay strength of the local soil-land surface system, how long it needs to take to make the influence of precipitation reduced to half. For example, in the places where the time of reduced-to-half is a week, the a can be taken as 0.9. With this value for the parameter, the influence of precipitation will be reduced to a quarter after 2 weeks and 1% after a month and half. This is in general consistent with the statement that the anomaly of soil moisture caused by a heavy rain or a dry period may take weeks to months to dissipate (Mahanama and Koster 2003).

For a specific location or area, the temporal variability of the flood and drought can be studied by using the index WAP. Compared with CRP, the relative flood and drought index WAP is less affected by the parameter a. When a increases, a^n increases for the earlier days but (1 - a) decreases, hence the weight w_n does not change much. Moreover, based on the change rate of w_n with a derived from (9)

$$\frac{dw_n}{da} = \left(\frac{1-a}{a} n - 1 \right) a^n , \tag{10}$$

the change rates are negative in the recent days when $n < a/(1 - a)$ (*e.g.*, $n < 9$ for $a = 0.9$) but are positive in the earlier days when $n > a/(1 - a)$ (*e.g.*, $n > 9$ for $a = 0.9$). Thus, overall, the WAP does not change much with a. This property has been demonstrated in Figure 2. The difference of the curves of WAP with different a (0.8 and 0.9) is that the day-to-day variation of WAP with l arger a is smoother than that with smaller a. Nevertheless, the seasonal cycles of WAP are very close and can give the same results in detecting the flood and drought, including their start, duration, breaks, and strength.

The a of being 0.9 is therefore suggested for the general use of the index WAP. Taking the truncation precision c as 1%, N is calculated to be 44. The index thus becomes

$$WAP = 0.1 \sum_{n=0}^{44} 0.9^n P_n . \tag{11}$$

d. Determining the start, duration, and strength

Zeng and Lu (2004) designed a method to determine objectively the monsoon onset and retreat dates from seasonal cycles of precipitable water with certain criteria. A similar approach can be used to determine the start and end dates (and thus the duration and strength) of the flood and drought from seasonal cycles of WAP. For a given location or area, the daily precipitation data of the 26 years, for example (as in Figure 2), are first converted into daily WAP values that include 25 years. The maximum and minimum of the WAP of each year, and thus their averages over the 25 years, are then calculated. The difference ranging from the averaged minimum to the averaged maximum can be divided into several grades, representing a classification from the severest drought to the severest flood. The start and end dates of the flood and drought for a specific severity grade can be determined with proper criteria, *e.g.*, when the daily WAP exceeds (is lower than) the grade for 3 consecutive days for the start (end). The strength of the flood or drought averaged over the determined flood or drought duration can be calculated. With the start, duration, and strength of flood and drought determined for each year, their interannual variabilities and long-term changes can be studied. By using this methodology, which has been partly applied in Figure 2, evaluations for different regions of the globe will be presented separately.

3. Summary and discussions

The SPI is commonly used for detecting flood and drought. When it is calculated, a timescale longer than a month needs to be selected first. The days in the period of the scale are treated equally with precipitation simply averaged over the period, while the precipitation in the days before the period is not considered. Because of these, the SPI can determine only the general flood and drought situation of a long period (*e.g.*, several months or years) but cannot be used for short scales (*e.g.*, a week or less). However, the operational monitoring and decision-making do require an index to measure the daily flood and drought extent. With a daily index, the start, duration, and strength of the flood and drought of a year can be determined, and their interannual variabilities and long-term changes as well as the associated mechanisms can be studied. As an objective state of the land-atmosphere system, the flood and drought extent should have an instantaneous value and not just have an overall condition of a long period.

Physical considerations of developing such an index for measuring daily flood and drought extent is provided in this note with a simple physical model. It uses only precipitation as the SPI does, but it also considers the demands of the water balance as the PDSI does. The principle of the physical model is that the flood extent is forced by precipitation, but dissipated by the demands of the water balance. Among the demands, runoff, groundwater flow, and percolation can be important during or just after a precipitation, while evapotranspiration can be important in the later dry time. What differs from the PDSI is that in this study, the overall dissipation or decay effect of these demands is represented with a parameterization of the flood extent. To be simple, but still fairly reasonable, the parameterization is currently taken as a linear form. The defined index, based on the solution of the physical model, has two forms, and both are easy to calculate.

The cumulation of reduced precipitation (CRP) can be used to analyze the spatial variability of flood and drought extent. The value of CRP can be affected by parameter *a*, the contribution strength of the earlier day precipitation. For meteorological flood and drought, which stress precipitation and the general effect of the demands, the parameter can be identical for different locations. If local hydrological and geographic conditions need to be considered for each specific location, the parameter can be given empirically based on the local characteristics, since the daily flood and drought extent is what needs to be quantified.

The advantage of the weighted average of precipitation (WAP) is that, as shown both analytically and from the observation, its value is less affected by the parameter *a*. The WAP measures the relative flood and drought extent as the SPI does, and can be used to evaluate the temporal (*e.g.*, seasonal and interannual) variations of the flood and drought extent of a specific location or area, which is more important than the spatial comparison of the extent. The SPI makes simple average of precipitation, so a timescale is required. The WAP makes weighted average of precipitation with the weight decreasing with the number of the days past, hence no timescale is needed. Figure 2 shows that the results of the start, duration, and strength of flood and drought and their year-to-year variations calculated with two different values of *a* are very close. The *a* of being 0.9 is finally suggested for the calculation of the WAP.

Modifications could be made in future studies to the current forms of the index to better reflect the flood and drought situations in places with different hydro-climate and geographical conditions. A relatively complicated parameterization *D*(*f*) may be developed, with knowledge of the local characteristics of a specific place, to better represent the dissipation effect of the demands. The physical model then becomes $df/dt = D(f) + P$, and this equation can be solved numerically. Modifications may also be made to the expression of the WAP with designing a new weight *W*(*n*), and the index is then expressed as $WAP = \sum_{n=0}^{N} W(n) P_n$.

The precipitation in cold season of some areas may be in form of snow, partially or totally, and later the snowmelt can contribute with rainfall to enhance the flood extent. It is better to replace the precipitation in the index with the combination of rainfall and snowmelt. This change may have some influence to the values of the index in snow and snowmelt seasons, but has little influence during the warm season. The physical model of this study can also be applied to the scale less than a day to study the diurnal variation of the flood extent, which may be large when there are thunderstorms.

The purpose of this note is to provide the physical basis of the daily flood and drought index and the general methodology of using the index to determine the start, duration, and strength of flood and drought from an example as well as their interannual variabilities and long-term changes. More calculations and detailed analyses will be presented in the follow-on studies for different regions of the globe.

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