

# A Multi-Conserving Finite Difference Scheme for NCEP GFS on a Generalized Vertical Hybrid Coordinate

By Hann-Ming Henry Juang



**Editorial Note:** Two years ago, Dr. Kevin Trenberth wrote a letter to NWS Director, attributing some stratospheric problems identified in the NCEP analysis to the inherent weakness of sigma coordinate employed by the NCEP model (Trenberth 2002) and recommending NCEP/EMC to change its model vertical coordinate to a hybrid system. Instructed by the director, OST coordinated an internal discussion on this issue and replied to Dr. Trenberth positively on behalf of the director. Tremendous efforts have been made by NCEP/EMC scientists in collaboration with outside research community since then. Here is an article on the progress of this important model development, which keeps the NWS promise.

A discretization of a hydrostatic primitive equation global atmospheric model on spherical and generalized hybrid vertical coordinates is described in NCEP Office Notes 445. The discretization in the horizontal using a spectral method with spherical transformation is as the same as used in NCEP global model, only the vertical discretization is illustrated (Illus. 1).

**A specific form for generalized hybrid**

$$\hat{p}_k = \hat{A}_k + \hat{B}_k p_s + \hat{C}_k \left( \hat{T}_{ik} / \hat{T}_{0k} \right)^{C_p / R_d}$$

where

$$\hat{A}_{K+1} = \hat{B}_{K+1} = \hat{C}_{K+1} = 0$$

$$\hat{A}_1 = \hat{C}_1 = 0$$

$$\hat{B}_1 = 1$$

For sigma-pressure, let  $\hat{C}_k = 0$  for all levels

- $\hat{B}_k$  - decrease from 1 to 0 at low layers
- $\hat{A}_k$  - increase from 0 to be pressure where B=0, then decrease to zero.

For sigma-theta, let  $\hat{A}_k = 0$  for all levels

- $\hat{B}_k$  works the same, and  $\hat{C}_k$  works as  $\hat{A}_k$  in sigma-p

*A description of the current generalized vertical hybrid coordinates is shown with three variables, A, B, and C. At any given model layers during integration, all these three variables have to fix as constants. While A=C=0, it is pure sigma coordinate. While either A or C is non-zero, it is hybrid coordinates.*

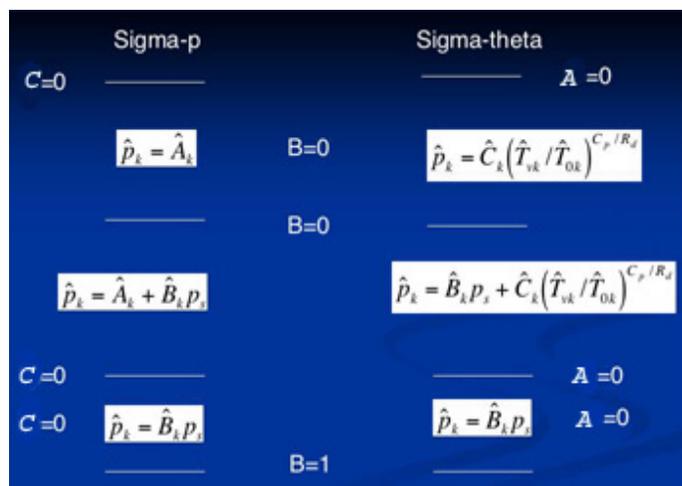
**Illustration 1**

Energy, entropy, and angular momentum conservation are used as constraints to discretize the vertical integration by finite difference scheme. The entire atmosphere is divided into several layers (Illus. 2); only pressure and vertical flux are specified at the interfaces, and other variables such as horizontal wind, temperature, specific humidity and specific amount of tracers are specified at each layer. Conservation is a constraint that requires the pressure at each layer to be averaged by the pressures at the immediate neighbor interfaces (the one above and one below a given layer). Since pressures are not in a logarithmic form, the relationship for pressure between layers and interfaces becomes simple, and with pressure equation not in logarithmic form, it provides mass conservation as an extra.

Due to the generalized vertical coordinate, vertical flux is solved by applying local changes in the pressure and virtual temperature equations to the definition of the vertical coordinate. It solves vertical fluxes at all interfaces by a simple algebraic equation through tri-diagonal inversion. For the sake of time splitting between dynamics and physics processes, the vertical flux obtained in the dynamics is without local change on isentropic surface, but the vertical advection is required in the model physics. The forward-time un-centered semi-implicit time integration scheme is also given by this finite difference scheme in generalized vertical coordinates under vertical profiles of reference temperature and pressure.

A specific definition of a generalized hybrid coordinate, including sigma, pressure and isentropic surfaces, is introduced to define pressure at interface. It is given by surface pressure and virtual temperature. These

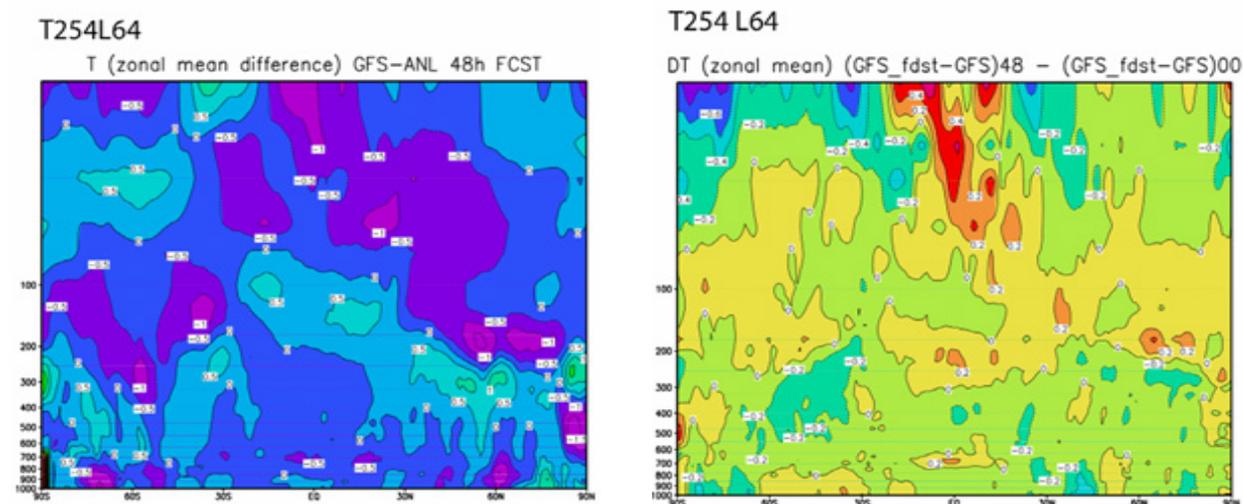
modifications of the generalized form provide computational saving, instead of solving for the pressure at all interfaces, only the surface pressure equation is needed. Though the elements in the matrixes for semi-implicit computations become more complicated than those in the generalized pressure equation at all levels, the computing time is not increased because the matrixes are of the same degree; also two matrixes reduce to vector computation only because the surface pressure is solved instead of pressure at all the interfaces.



A schematic plot shows two kinds of hybrid coordinates. The left one is sigma-pressure hybrid coordinate, the right one is sigma-theta coordinate. In both case, B=1 at ground surface, then decreases to be zero at certain layers, above that layer B=0 and another one describes the model layer, either isobaric surfaces (by A with C=0) or isentropic surfaces (by C with A=0).

**Illustration 2**

The preliminary results indicate the differences among all types of coordinates, sigma, sigma-pressure and sigma-theta, are small, but a tendency to have improved performance is shown in hybrid coordinates of sigma-pressure and sigma-theta. A bias of zonal mean temperature used to be existed in current operational model. The hybrid coordinates results, especially on sigma-theta, show an indication of a correction over this bias (Fig. 1). The noises by divergence or vertical motion over topper layers are smaller or eliminated on hybrid coordinates. More works have to be done to provide proofed impacts before an operational implementation.



**Figure 1** Zonal mean temperature 48-hour forecast bias seen in the operational GFS (left) and the GFS with sigma-theta hybrid vertical coordinate implemented (right). The differences, as compared between the two figures, indicate a bias correction for the T254L64 GFS.

Correspondence to: Dr. Hann-Ming Henry Juang, NOAA/NWS/Environment Modeling Center  
E-mail: Henry.Juang@noaa.gov