FLOOD ROUTING MODELS AND THE MANNING n

by

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ABSTRACT

Flood routing models based on the one-dimensional Saint-Venant equations often utilize the Manning n to represent flow resistance. The selection of n values for use in flood routing models are discussed with regards to (1) the use of historical flood observations and (2) when no observed flow data are available. Some typical variations of the Manning n with unsteady flows for some reaches of a few principal rivers in the United States are presented. These were obtained through calibration of the National Weather Service (NWS) Dynamic Wave (DWOPER) flood routing model. Also, numerical and analytical sensitivity studies are presented which illustrate the effect of the uncertainty associated with selected n values on the routing model's predicted stages. Also, the sensitivity of predicted stages to long term variation of the Manning n for the lower 300 miles of the Mississippi River is summarized. Finally, a methodology is presented for including within flood routing models the combined frictional effects of dynamic alluvial bed-forms and overbank vegetation.

INTRODUCTION

In the United States, many flood routing models based on the one-dimensional Saint-Venant equations of unsteady flow, e.g. Fread (1978, 1985, 1988), Schaffranek, et. al, (1981), Johnson (1974), Amein and Fang (1970), Garrison et. al, (1969), and Baltzer and Lai (1968) utilize the Manning n as a component of the friction slope (S_f) term within the conservation of momentum equation to account for resistance to flow, i.e.,

\[ S_f = \frac{Q|Q|n^2}{(2.21A^2R^{4/3})} = \frac{Q|Q|}{K^2} \]  \hspace{1cm} (1)

where: \[ K = 1.486 A R^{2/3} / n \]  \hspace{1cm} (2)

in which Q = discharge, A = wetted cross-sectional area, R = hydraulic radius, and K = the conveyance factor. The resistance to flow is parameterized by the Manning n which represents the effect of roughness elements made up of bank and bed particles as well as form losses attributed to dynamic alluvial bed-forms and vegetation of various types (grass, shrubs, field crops, brush and trees) located along the banks and overbanks (floodplain). Also, small eddy losses due to mild expansion/contraction of cross-sectional reaches and river bend losses are often included as components of the Manning n.

SELECTION OF THE MANNING n

Although the literature provides some guidance for selecting the Manning n, it can only be determined accurately for river applications by computing it using the Manning equation, i.e.,
in which \( Q \), \( A \), \( R \), and \( S_p \) are measured for a particular flow through a particular reach of a particular river. As the magnitude of flow changes, the value of the Manning \( n \) also changes. Some other conditions can result in different \( n \) values for the same flow. Some of these are: (1) change of season which affects extent of vegetation, (2) change of water temperature which affects bed-forms in some alluvial rivers, (3) ice cover effects, (4) man-made channel changes, and (5) the sequence of a flood event inundating a floodplain susceptible to deformation of the vegetative resistance.

When the flow is unsteady as during the passage of a flood hydrograph, a technique for computing the Manning \( n \) other than via Eq. (3) was introduced by Fread and Smith (1978). This consists of an automatic calibration algorithm which is an option within the NWS DWOPER flood routing model (Fread, 1978, 1985). Manning \( n \) values, which are delineated as a function of discharge, are automatically computed for the range of flows encompassing the entire observed (historical) flood hydrograph such that the differences between water surface elevations computed by the model and observed stage hydrographs from level recorders are minimized. The algorithm for efficiently accomplishing this is applicable to a single multiple-reach river or dendritic system of rivers consisting of a main-stem river and its principal tributaries.

Also, the Manning \( n \) for the range of flows associated with previously observed floods may be selected via a trial-and-error calibration methodology. With observed stages and flows, preferably continuous hydrographs from a previous large flood, the DWOPER model can be used to determine the \( n \) values as follows: (1) use the observed flow hydrograph as the upstream boundary condition and select an appropriate downstream boundary (an observed stage hydrograph at the downstream boundary could be used if available); (2) estimate the Manning \( n \) values throughout the routing reach; (3) obtain computed \( h \) and \( Q \) from the solution of the Saint-Venant equations; (4) compare the computed elevations with the observed elevations at the upstream boundary and elsewhere; (5) if the computed elevations are lower than the observed, increase the estimated \( n \) values; or if the computed elevations are higher than the observed, decrease the estimated \( n \) values; (6) repeat steps (3) and (4) until the computed and observed elevations are approximately the same. The final \( n \) values are sufficient for the range of flows used in the calibration; however, the \( n \) values for flow elevations considerably exceeding the observed must be estimated as described below. The calibrated \( n \) values, however, provide an initial estimate from which the unknown \( n \) values may be extrapolated or ultimately approximated.

In the absence of observed flows and water elevations, selection of the Manning \( n \) should reflect the influence of bank and bed materials, channel obstructions, irregularity of the river banks, and especially vegetation. The latter may cause the \( n \) values to vary considerably with flow elevation, i.e., the \( n \) value may be considerably larger for flow inundating the floodplain than for flow confined within the channel bank. This is due to the presence of field crops, weeds, brush, scattered trees, or thick woods located in the floodplain. Also, the \( n \) value may be larger for small floodplain depths than for larger depths. This can be due to a flattening of the brush, thick weeds,
or tall grass as the flow depths and velocities increase. This effect may be reversed in the case of thick woods where, at the greater depths, the flow impinges against the branches having leaves rather than only against the tree trunks. Seasonal influences (leaves and weeds occur in summer but not in winter) may also affect the selection of the Manning n. Basic references for selecting the Manning n may be found in Chow (1959) and Barnes (1967). Also, two recent reports from the USGS should be considered in selecting n values, i.e., Arcement and Schneider (1984) for wooded floodplains and Jarrett (1984, 1985) for relatively steep \(0.002 < S_o < 0.040\) streams with cobble/boulder beds. Both of these also provide general methodologies quite similar to that given by Chow (1959) for selecting the n value to account for the various factors previously mentioned. Arcement and Schneider (1984) also consider the effects of urbanization of the floodplain. Another methodology which estimates the Darcy friction factor \((f)\) for floodplain flows is described by Walton and Christenson (1980). The Darcy \(f\) is related to the Manning \(n\) as follows:

\[
n = 0.0926 f^{0.5} D^{0.17}
\]

in which \(D\) = the hydraulic depth.

**Dam-Break Floods**

The flow observations used in developing the Manning n predictive methodologies have been confined to floods originating from rainfall/snowmelt-runoff. The much greater magnitude of a dam-break flood produces greater velocities and results in the inundation of portions of the floodplain never before inundated. The higher velocities will cause additional energy losses due to temporary flow obstructions formed by transported debris which impinge against some more permanent feature along the river such as a bridge or other man-made structure. The dam-break flood is much more capable than the lesser runoff-generated flood of creating and transporting large amounts of debris, e.g., uprooted trees, demolished houses, vehicles, etc. Therefore, the Manning n values often need to be increased in order to account for the additional energy losses associated with the dam-break flows such as those due to the temporary debris dams which form and then disintegrate when ponded water depths become too great. The extent of the debris effects, of course, is dependent on the availability and amount of debris which can be transported and the existence of man-made or natural constrictions where the debris may impinge behind and form temporary obstructions to the flow.

**VARIATION OF n WITH FLOW**

The Manning n varies with the magnitude of flow. This is due to resistance caused by vegetation (brush, trees, etc.) located along the banks and overbanks of the river. As the flow increases and more portions of the bank and overbank become inundated, the vegetation located at these elevations can cause an increase in the total resistance to flow. This type of variation of the Manning n with flow is shown in Fig. 1 for portions of the n vs. flow relations for the Baton Rouge-Donaldsonville reach of the lower Mississippi River and the Shawneetown-Fords Ferry reach of the lower Ohio River. However, some major rivers in the U.S. often tend to show the opposite n vs. flow effect, i.e., the n values decrease with increasing discharge. This is the
case where the increase in the overbank flow area is relatively small compared to the increase of flow area within bank as with wide rivers with levees situated closely along the natural river banks. This type of $n$ vs. discharge effect is illustrated in Fig. 1 for the following: (1) the Donaldsonville-New Orleans reach of the lower Mississippi River, (2) the Cairo-Caruthersville reach of the middle Mississippi River, (3) the Chester-Cairo reach of the upper Mississippi River, and (4) the Kentucky Dam-Ohio River reach of the lower Tennessee River. Both of the above $n$ vs. discharge relations are evident in the Baton Rouge-Donaldsonville and Shawneetown-Fords Ferry reaches. The $n$ is fairly consistent with flow for the range of flows between 80,000 and 230,000 cfs for the Warrendale-Tongue pt. reach of the lower Columbia River.

![Diagram of Manning $n$ vs. discharge](image)

**Fig. 1. Variation of Manning $n$ with discharge**

The various $n$ vs. discharge relations shown in Fig. 1 were determined from the automatic calibration procedure in the NWS DWOPER flood routing model applied to the following river systems:

**Lower Mississippi**

A schematic of the 292 mi reach of the lower Mississippi River consisting of eight water level recorders is shown in Fig. 2. The discharge is known at the most upstream station. This reach of the lower Mississippi is contained within levees for most of its length. The average channel slope is an extremely mild 0.0000064. The discharge varies from low flows of about 100,000 cfs to flood discharges of over 1,200,000 cfs. A total of 25 cross sections located at unequal intervals including the locations of the level recorders were used in the computations. The effectiveness of the optimization is represented by the root-mean-square (rms) error between the computed and observed hydrographs at each level recorder such as that shown for Donaldsonville in Fig. 3. The average rms value for seven level recording stations shown in Fig. 2 is 0.37 ft, which is only 2.4 percent of the total change in water elevation.
Ohio-Mississippi

Fig. 4 is a schematic of dendritic river system consisting of 393 miles of the Mississippi, Ohio, Cumberland, and Tennessee Rivers with a total of 16 water level recorders and discharge measurements at the most upstream stations on each of the four rivers. The channel bottom slope is mild, varying from about 0.000047 to 0.000095. Each branch of the river system is influenced by backwater from downstream branches. Total discharge through the system varies from about 120,000 cfs to flood flows of 1,700,000 cfs. A total of 45 cross-sections located at unequal intervals were used in the computations. An example of a computed and observed stage hydrograph for Cairo is shown in Fig. 5. The average rms error, for 13 level recording stations shown in Fig 4, is
0.62 ft, which represents 2.5 percent of the total change in water elevation during the flood.

**Lower Columbia**

The lower 128 miles of the Columbia River and the lower 24.4 mile reach of the Willamette River have a very flat bottom slope (0.000011), and the flows are quite affected by the tide from the Pacific Ocean. A schematic of the reaches modeled are shown in Fig. 6. The tidal effect extends as far upstream as the tailwater of Bonneville Dam during periods of low flow. Reverse flows can occur as far upstream as Vancouver. A total of 25 cross sections located at unequal distance intervals were used in the computations. The average rms error for a 3-day low flow period at eight level recording stations shown in Fig. 6 was 0.37 ft or 6.9 percent of the total change in water elevation. A comparison of computed and observed water elevations at Portland is shown in Fig. 7.

![Schematic of lower Columbia River System](image)

**Fig. 6.** Schematic of lower Columbia River System

**Fig. 7.** Stage hydrograph at Portland

**EFFECTS OF UNCERTAINTY OF n**

The Manning n values used to represent frictional resistance of floods in rivers always have some degree of uncertainty associated with them. This affects the stages (water surface elevations) predicted by flood routing models. Fig. 8 illustrates the effect on the Donaldsonville stage hydrograph when the Manning n relation for the lower Mississippi reach from Baton Rouge to New Orleans is changed. The effect is a change in stage of ±2 ft or approximately ±10 percent of the peak stage. An increase in n causes an increase in stage at Donaldsonville and a decrease in n causes a decrease in stage. Although a one to one relationship between a change in Manning's n and the resulting change in the computed stage does not exist for the Donaldsonville gage; nevertheless, the effect of altering n is quite significant.

When the n values vary from reach to reach along a river, as found to be the case for the lower portion of the lower Mississippi River, changes in the n values for any single reach produce varying changes in the computed stages
at all points along the river. This effect is shown in Fig. 9. In the case of a 20 percent increase in the friction for the Donaldsonville-New Orleans reach, the rms variation in the stage hydrographs changes from positive to negative depending upon the location of the gage in question with respect to the reach of river for which the n values are increased. The most significant change is an increase in the stages at locations a short distance upstream and downstream of Donaldsonville; however, the effect vanishes at locations far upstream and downstream of Donaldsonville. Also, it should be noted that the variation of the rms of the stage hydrographs changes from a positive effect to a negative effect at a particular location within the reach in which n is increased. Quite similar but opposite effects are produced in the rms of the stage hydrographs when the friction of the Donaldsonville-New Orleans reach is decreased by 20 percent.

Several historical floods from the period 1959-71 were simulated with the DWOPER model using the calibrated Manning n values obtained from the 1969 flood. An example of simulated vs. observed stages is shown in Fig. 3 for the 1966 flood. Average rms errors for all seven stations for each of the simulated floods are shown in Table 1. The average rms error for all the floods was 0.47 ft. This compares with 0.25 ft for the calibrated flood of 1969, indicating that for this reach of the Mississippi there is not a significant change in the channel roughness from one flood event to another. However, in river reaches where sediment transport effects are greater or where floods occur at different seasons when vegetative and temperature effects are significant, the variation of the n values from flood to flood can be significant.

The Manning Equation (3) can be used analytically to approximate the effect of uncertainty in the Manning n on the computed flow depth (h) as follows (Fread, 1981). Let the river cross-sectional top width (B) and area (A) be represented by a power function using a scale factor (k) and a shape factor (m), i.e., m = 0 (rectangular), m = 0.5 (parabolic) m = 1.0 (triangular), and m > 1 represents a triangular section shape with sides which flatten to the
Table 1: Summary of flood simulations in Lower Mississippi River (Red River Landing to Venice) for the years 1959–1971

<table>
<thead>
<tr>
<th>Year</th>
<th>Average r.m.s. error (ft.)</th>
<th>Peak discharge (1000 cfs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959</td>
<td>0.62</td>
<td>750</td>
</tr>
<tr>
<td>1960</td>
<td>0.31</td>
<td>850</td>
</tr>
<tr>
<td>1961</td>
<td>0.47</td>
<td>1220</td>
</tr>
<tr>
<td>1962</td>
<td>0.61</td>
<td>1155</td>
</tr>
<tr>
<td>1963</td>
<td>0.38</td>
<td>905</td>
</tr>
<tr>
<td>1964</td>
<td>0.41</td>
<td>1140</td>
</tr>
<tr>
<td>1965</td>
<td>0.44</td>
<td>1100</td>
</tr>
<tr>
<td>1966</td>
<td>0.38</td>
<td>1090</td>
</tr>
<tr>
<td>1967</td>
<td>0.38</td>
<td>700</td>
</tr>
<tr>
<td>1968</td>
<td>0.36</td>
<td>980</td>
</tr>
<tr>
<td>1969*</td>
<td>0.25</td>
<td>1065</td>
</tr>
<tr>
<td>1970</td>
<td>0.91</td>
<td>1080</td>
</tr>
<tr>
<td>1971</td>
<td>0.46</td>
<td>940</td>
</tr>
</tbody>
</table>

*Calibrated

Fig. 10. Error in stages due to error in Manning n for steady flow horizontal as the depth increases from the vertex of the triangle (the degree of flattening increases as m exceeds unity). Thus, a power function representation of the cross-sectional properties is given by the following:

\[ B = kh^m \]  
\[ A = kh^{m+1} \]  
\[ h = (Q/a)^b \]

where:

\[ a = 1.49 \sqrt{S_f} \frac{k}{[n(m+1)]^{5/3}} \]

\[ b = 3/(3m + 5) \]

In which \( S_f \) is the friction slope which for steady, uniform flow may be approximated as the channel bottom slope.

The effect that the error in the frictional resistance or the Manning \( n \) has on the flood depth can be obtained by using Eq. (7). Thus,

\[ \frac{h_e}{h} = (n_e/n)^b \]

in which the subscript \((e)\) designates quantities possessing some error. Since the fitting coefficient \( m \) is present in Eq. (10) via the term \((b)\), the general shape of the cross section as determined by \( m \) must be considered. An expression for the percent error in the Manning \( n \) (\( E_n \)) may be developed in terms of the ratio \( h_e/h \), i.e.,

\[ E_n = 100 \left( \frac{n_e}{n} - 1 \right) \]

If \( E_n \) is plotted against \( E_h \) (percent error in computed stage) for a range of cross-sectional shapes as shown in Fig. 10, it is apparent that the relationship is nonlinear and that errors in the Manning \( n \) (\( E_n \)) are dampened when transformed into errors in flow depth (\( E_h \)). The extent of dampening is
directly proportional to the channel slope or the $n$ coefficient of Eqs. (5-6). The error dampening characteristic is beneficial for the prediction accuracy of computed depths.

Errors in the Manning $n$ will also have a significant effect on the kinematic celerity (approximate propagation speed) of the flood wave. Using Eq. (10) the following celerity ratio can be developed:

$$ c_e/c = (n_e/n)^{2b/3} - 1 $$

Eq. (12) indicates that errors in the Manning $n$ will produce damped errors in the wave celerity. For example, if $E_n = +50\%$ and $m = 1.0$, the error in the wave celerity would be only $-25\%$. Also, the influence of the cross-section shape factor $(m)$ is very weak in Eq. (12).

SIMULATING COMBINED SAND-BED/VEGETATIVE FRICTION

The frictional effects in some rivers with substantial overbank flows is controlled by two quite different mechanisms. One of these is the frictional effects caused by the presence of a mobile sand-bed which consists of two components: (1) skin friction as related to the sand grain size, and (2) form roughness due to the dynamic bed-forms (dunes, ripples, plain bed, antidunes). The second mechanism is the vegetation (trees, shrubs, field crops, brush, etc.) located along the banks and overbanks. Flood routing models must simulate the frictional effects throughout the entire range of flows from low flow, confined within banks where the frictional effects are due solely to sand-bed effects, to the peak flows where the frictional effects are divided between the sand-bed effects and the vegetative effects. A methodology for simulating this combined frictional effect is being investigated at the NWS Hydrologic Research Laboratory. The sand-bed effect is being simulated with the procedure of Brownlie (1983) and the vegetative effect with an empirical $n$ vs. $Q$ relation developed via the automatic calibration technique in the NWS DWOPER flood routing model.

REFERENCES


