II.4-STAGEQ  STAGE-DISCHARGE CONVERSION FOR A SINGLE VALUED RATING CURVE

Introduction

This Section describes the methods available for converting a given stage or discharge to the associated discharge or stage using interpolation or extrapolation of a single-valued Rating Curve.

A single-valued Rating Curve is one that has the same stage-discharge relationship when the river level is rising or falling as illustrated in Figure 1.

Either linear or logarithmic interpolation can be used to determine stage from discharge or vice-versa when the given value lies within the range of the defined Rating Curve points. When the given value lies outside the specified range then extrapolation is necessary. Extrapolation can be done using linear, logarithmic or hydraulic methods to extend the Rating Curve.

Linear Interpolation/Extrapolation

The basic equations used for linear interpolation and extrapolation are as follows:

1. Determine the slope (a) of the Rating Curve from the two points that bracket the given value in the case of interpolation or the two points closest to the given value in the case of extrapolation. The point with the largest values is designated $H_u$, $Q_u$ and the one with the smallest values is $H_l$, $Q_l$. The slope is computed as:

$$ a = \frac{Q_u - Q_l}{H_u - H_l} $$  (1)

2. To convert a given stage ($H'$) to discharge ($Q'$):

$$ Q' = Q_l + a(H' - H_l) $$  (2)

3. To convert a given discharge ($Q'$) to stage ($H'$):

$$ H' = H_l + \frac{1}{a}(Q' - Q_l) $$  (3)

Logarithmic Interpolation/Extrapolation

The basic equations used for logarithmic interpolation and extrapolation are as follows:

1. Determine the logarithmic slope (a) of the Rating Curve from the two points that bracket the given value in the case of interpolation or the two points closest to the given value in the case of extrapolation. The point with the largest values is designated $H_u$, $Q_u$ and the one with the smallest values is $H_l$, $Q_l$. The slope is computed as:

$$ a = \frac{\log Q_u - \log Q_l}{\log H_u - \log H_l} $$  (4)

2. To convert a given stage ($H'$) to discharge ($Q'$):

$$ Q' = Q_l \cdot e^{a(H' - H_l)} $$  (5)

3. To convert a given discharge ($Q'$) to stage ($H'$):

$$ H' = H_l + \frac{1}{a}\left(\log \frac{Q'}{Q_l}\right) $$  (6)
the case of extrapolation. The point with the largest values
is designated $H_u$, $Q_u$ and the one with the smallest values is $H_l$, $Q_l$. The slope is computed as:

$$ a = \frac{\log Q_u - \log Q_l}{\log H_u - \log H_l} \quad (4) $$

2. To convert a given stage ($H'$) to discharge ($Q'$):

$$ \log Q' = \log Q_u + a(\log H' - \log H_u) \quad (5) $$

$$ Q' = e^{\log Q'} \quad (6) $$

3. To convert a given discharge ($Q'$) to stage ($H'$):

$$ \log H' = \log H_u + \frac{1}{a}(\log Q' - \log Q_u) \quad (7) $$

$$ H' = e^{\log H'} \quad (8) $$

If $Q_l$ is equal to zero then $Q_l$ is set to 0.0001 CMS to allow for
logarithmic calculations. In addition stages less than the specified
minimum allowable value ($H_m$) are not used in the computations or
generated as the result.

If the Rating Curve does not plot as a straight line on logarithmic
paper then the interpolation and extrapolation results will not be
correct. When this occurs, an offset value can be used to shift the
Rating Curve so that it does plot as a straight line on logarithmic
paper. The offset factor ($O_f$) is subtracted from all the stage values
in equations 4, 5 and 7. When an offset factor is used Equation 8
becomes:

$$ H' = e^{\log (H' - O_f)} + O_f \quad (9) $$

In some cases different offset factors need to be applied to
different ranges of stage in order to straighten out the logarithmic
Rating Curve plot. The stages above which user specified offset
factors apply must be defined points on the Rating Curve. Also a
given offset factor must be less than the minimum stage to which it
applies ($H_f$) so that the shifted stages will be greater than zero.

Offset factors can also be used to allow negative stages to be used
in the logarithmic interpolation/extrapolation equations. In this
case negative offset factors insure that positive stage values are
used in the equations. If the user does not specify offset factors
such that all stages used in the calculations are positive then the
program automatically computes a shift factor ($S_f$) to produce the same
results. $S_f$ is used exactly like $O_f$ in equations 4, 5, 7 and 9 except
$S_f$ is added to all stage values (i.e. $S_f=-O_f$). The rules used to
determine $S_f$ and the stage below which it is applied ($H_s$) are as follows:

1. If zero flow is a defined point ($H_o$, $Q_o$) on the Rating Curve then:
   a. If specified $H_m$ is less than $H_o$ then set $H_m=H_o$
   b. If $H_o>0.0$ or $H_o=\min(H_f)$ then:
      
      $S_f=0.0$
      
      and
      
      $H_s=H_o$
   c. If $H_o\leq0.0$ and $\min(H_f)>H_o$ then:
      
      $S_f=0.01-H_o$
      
      and
      
      $H_s=\min[\min(H_f),H_p]$
      
      where $H_p$ is the smallest positive stage value defined

2. If the zero discharge point is not included in the Rating Curve definition then:
   a. If $H_m$ and the stage at the first defined point on the Rating Curve ($H_1$) are both greater than zero then:
      
      $S_f=0.0$
      
      and
      
      $H_s=H_o$
      
      where $H_o$ is computed by using $Q_o=0.0001$ CMS
   b. If $H_m$ or $H_1\leq0.0$ then:
      
      $S_f=0.01-\min[H_m,H_1]$
      
      and
      
      $H_s=\min[\min(H_f),H_p]$
   c. If $H_m$ is less than the computed $H_o$ then:
      
      $H_m=H_o$

The shift factor will never be applied to the same range of stages to which an offset is applied. The shift factor is also only applied to the portion of the Rating Curve that needs to be shifted to avoid zero or negative values in logarithmic calculations. The shifted
portion of a Rating Curve most likely will not plot as a straight line on logarithmic paper. Therefore it is better off to specify offset factors to insure that reasonable results are obtained during extrapolation at the low end of the Rating Curve rather than to rely on the automatic shift procedure.

**Hydraulic Extension**

Hydraulic extension is used to determine the stage or discharge associated with a given discharge or stage, respectively, that lies beyond the range of the Rating Curve by examining the hydraulic properties of the stream cross-section.

The Manning equation is the basis of this method:

\[
Q = \frac{1}{n}S^{1/2}A^{5/3}/B^{2/3} = C\cdot A^{5/3}/B^{2/3}
\]  

(10)

where \(Q\) is the discharge  
\(n\) is the roughness coefficient  
\(S\) is the hydraulic slope  
\(A\) is the flow areas below the water surface  
\(B\) is the top width at the water surface  
\(C\) is:

\[
\frac{1}{n}S^{1/2}
\]

To determine \(Q\) given the stage the program examines the trend of the value \(C\) near the Rating Curve end point and extrapolates this value to the stage given. The cross-sectional area and top width are linearly interpolated or extrapolated internally. The discharge associated with the given stage is computed from Equation 10.

If the roughness coefficient \((n_2)\) above the uppermost elevation of the Rating Curve is entered then a composite roughness coefficient will be used for the hydraulic extension and is estimated from:

\[
n = \left[\frac{n_1^2 \cdot B_1 + n_2^2 \cdot B_2}{B}\right]^{1/2}
\]  

(11)

where \(B\) is the interpolated/extrapolated total topwidth, above the uppermost elevation of the Rating Curve, at the water surface elevation  
\(n\) is the composite roughness coefficient; applicable to total topwidth \(B\)  
\(B_1\) is the topwidth at the uppermost elevation of the Rating Curve  
\(n_1\) is the roughness coefficient (as determined from extrapolated 'C' values); applicable to topwidth \(B_1\)  
\(B_2\) is \(B - B_1\)
\( n_2 \) is the roughness coefficient above the uppermost elevation of the Rating Curve; applicable to topwidth \( B_2 \).

The discharge for the given stage is again computed using Equation 10.

To determine stage given discharge, a more complex solution technique is used as the flow area and top width are both functions of stage and a solution cannot be explicitly obtained. In this case a Newton-Raphson iteration technique is employed which uses trial values for stage in a function that accounts for the difference in the given discharge and the discharge that would be computed using the trial stage value. During each iteration a new trial stage value is computed that reduces the difference between the given discharge and the trial value discharge. Computation continues until the change in trial stage values from one iteration to the next is less than a preset value or until 50 iterations have been made, at which time a message indicating the occurrence of non-convergence is printed and the desired stage is found by linear or logarithmic extrapolation.

Hydraulic extension will be used only at the upper end of the Rating Curve. Linear or logarithmic extrapolation is used to extend the Rating Curve at the lower end.

A deficiency was detected in the hydraulic extension option in dealing with rapid change of channel topwidth. The impact becomes even more prominent in the case of a sudden presence of a very large channel topwidth with a very small flow area beneath it. Because the discharges in the hydraulic extension option are computed from the hydraulic radius and the area of an equivalent channel, the computed elevation-discharge relation could be erratic near and above the elevation where the rapid change of topwidth took place.

Discharges for the higher stages could be less than those for the lower stages, a result only possible in the loop rating option. A conveyance approach, which accounts for the channel flow and the flood plain flow separately, can be used to overcome these problems but is not currently available.
Figure 1. Stage-Discharge relationship for a single-valued Rating Curve

![Stage-Discharge Relationship](image)

- Stage
- Discharge